Differently dated dollars are different goods. A $1 bill today is not the same thing as $1 bill tomorrow.

The main idea here is that dollars in the future are less valuable than dollars now. Consider the following example:

> Suppose that the nominal interest rate is 10% (more later). Suppose you have $100, a present amount. You can put this $100 in the bank for one year, and at the end of the year, you will have $110. This means that $100 today has the same value as $110 next year. Or flipped around, $110 one year from now is worth $100 in present dollars. They are the same economic entity.

<u>Real interest rate</u>

We might find people who would be willing to trade $90 today in exchange for $100 next year. Here, the $90 today is a present amount or present value (P), and the $100 next year is a future amount or future value (X). If we do some math, we might find….

$X / P = \$100 / \$90 \approx 1.1 = 1 + 0.1$

People pay a premium to have dollars now (giving 100 in the future to get 90 now)
People are getting a discount for waiting for later availability (giving up 90 to get 100 later)

The real interest rate, r, is the premium for earlier availability of goods. It is usually measured in % per year, but can easily be adjusted for monthly rates, daily rates, etc.

The extra 0.1 (10%) is the premium on earlier availability in this example.

We can say then,

$X / P = 1 + r$, thus $r = 0.1$, so in this case, $r \approx 10\%$

<u>Adjustments/Complications for real world</u>

In a world with completely stable prices and no chance of default, the real interest rate would be the whole story. (X / P) would equal (1 + r.) However, in the real world, prices generally rise, and some people default. We need to adjust for:

1. Rising prices – historically in the US, prices are usually rising. Interest rates will take rising prices into consideration.
2. Default risk – there is a chance that after loaning money to someone, they will default on the loan (not pay it back). A premium will be charged to incorporate this added risk.

We will adjust r, the real interest rate for these complications. The result, **the nominal interest rate, R**, is

$R = r + \pi^e + \rho^d$

R = nominal interest rate
r = real interest rate
π^e = expected inflation
ρ^d = default premium

Thus, when considering the situation above ($90 vs. $100), we now know some of that 10% premium was really compensation for the fact that prices are rising and the chance of default. We will want to use the nominal interest rate when we do these calculations.

Thus, $X / P = 1 + R$. That is, we must use the nominal interest rate when doing present value calculations.

Suppose r =3%, π^e = 3%, and ρ^d = 4%.   Then R = 3% + 3% + 4% = 10%

Suppose r = 3% and π^e = 3%, and your credit card rate (R) =18%. Then ρ^d = 12%

Of course, ρ^d values will vary for people. If you have a good credit history, you will pay a smaller default premium. If you have a sketchy repayment history, you will have a higher default premium, and hence pay a higher nominal interest rate.

The interest rate you will observe in a contract, for instance your credit card bill, will always be a nominal interest rate. The 16.99% is a nominal interest rate. The 10% on your car loan is a nominal interest rate. The 7.75% on your Stafford loan is a nominal interest rate.

Note: unless otherwise specified, we will generally ignore the default premium. That is, we will assume that $\rho^d = 0$.

Also, $R = r + \pi^e$ (ignoring the default premium) is often called the Fisher equation, in honor of Irwin Fisher.

<u>The real interest rate is about goods</u>

Chad loans Dave $1000 for one year at 15% nominal interest rate (R = 15%). The price of a beer is $2 and everyone expects it to cost $2.20 next year ($\pi^e$ = 10%). At the end of the year, Dave will repay $1150.

Before we go right to the formula and calculate, let's look at this in terms of the number of goods that the $1000 and $1150 could have bought. Remember, its not so important how many dollars we have, but how many goods these dollars can purchase.

Originally, Chad had $1000. Chad could have bought 500 beers with his $1000

$1000 / $2 per beer = 500 beers

After the year went by, Chad will be repaid $1150, but in the meantime, the price has risen to $2.20. Chad can now buy 523 beers with his $1150.

$1150 / $2.20 per beer = 523 beers.

Chad has given up 500 beers today for 523 beers next year. **Thus, he has gained roughly 5% (23 additional beers from the original 500). This is what we mean by the real interest rate**. The real interest rate was roughly 5% (really closer to 4.6%, due to a slight approximation that you needn't be concerned with).

Dave however, gives up 523 beers next year for 500 beers today. Dave had to pay a premium (23 beers) to get earlier availability.

<u>What did our formula say?</u>

$R = r + \pi^e$ (ignoring default premium – this is called the Fisher Equation)

We knew the agreed on 15% as a nominal interest rate, and everyone expected inflation to be 10%. Thus,

15% = r + 10%   we can solve for r

r = 5%.

It is the real interest rate that is important, the 5% that represents the extra goods that can be consumed.

Expected inflation versus unexpected inflation

However, in the previous example, everyone agreed that expected inflation would be 10% and in fact, inflation turned out to be 10%. What happens if we have

a. unexpected inflation (higher inflation than expected)
b. unexpected deflation (lower inflation than expected)

Again, for simplicity, ignore the default premium.

$R = r + \pi^e$

The nominal interest rate (which we know, 15% for example) considers people's expectations about inflation. Thus we can calculate the ex-ante real interest rate (expected)

$r = R - \pi^e$

That is, with a nominal interest rate of 15% and expected inflation of 10%, the real interest rate (expected) would be 5%. That is, when we entered into the contract, we both expected that we would have a real interest rate of 5%.

However, it may actually be the case, ex-post (after the fact) that inflation was not equal to what people had expected. In this case, the ex-post real interest rate is

$r = R - \pi$ (where there is no subscript on the π, because we actually know this number).

Example, unexpected inflation

Suppose that actual inflation turned about to be 12% (higher than expected). This is unexpected inflation. What happens to the actual (ex-post) real interest rate? (The price of a beer is now $2.24)

$r = R - \pi = 15\% - 12\% = 3\%$ People expected an r of 5%, but it actually turned out to be 3%. (In our example, Chad's $1150 only buys 513 beers.)

Chad had expected to be paid back with enough money to buy 523, but now can only buy 513 beers so he is unhappy. Chad is sad. Dave however is giddy. Lenders (Chad) are worse off, borrowers (Dave) are better off.

Example, unexpected deflation

Suppose the actual inflation turned about to be only 8% (lower than expected). This is unexpected deflation. (Notice we still have inflation, but we still call this unexpected deflation because inflation was lower than expected). What happens to the actual (ex-post) real interest rate? (The price of a beer is now $2.16)

$r = R - \pi = 15\% - 8\% = 7\%$ People expected an r of 5%, but it actually turned out to be 7%. (In our example, Chad's $1150 now buys 532 beers).

Chad is now paid back with enough money to buy 532 beers so he is happy. Dave is miffed. Lenders are better off, borrowers are worse off.

Who wins and who loses – a summary and a better way to remember?

If you are loaning money, you want a high r.
If you are borrowing money, you want a low r.

Unexpected inflation lowers the actual r, so it helps borrowers and harms lenders.

Unexpected deflation raises the actual r, so it harms borrowers and helps lenders.
I always forget this, but I remember that I am usually a borrower (student loans). Thus, I would be made better off by unexpected inflation.

We'll talk about the Wizard of Oz sometime in class, but as a preview, some say that the story of the Wizard of Oz is an allegory for monetary policy as it relates to the political scene of the 1896 presidential election. William Jennings Bryant proposed changing monetary policy in a way that would have increased the inflation rate. Farmers, are typically people that have borrowed money, and thus, could benefit from this unexpected inflation. Do you think Bryant did well in the midwestern states?

Some terminology, as a reminder

Unexpected inflation, technically, is $\pi > \pi^e$. It is faster than expected inflation. Think unexpectedly high inflation.

Unexpected deflation, technically, is $\pi < \pi^e$. It is slower than expected inflation. Think unexpectedly low inflation.

Present Value formulas

If I have \$100 in the bank and I put it in the bank at an interest rate of 10%, I can convert the \$100 today into \$110 next year.

On the other hand, if I know I will get \$110 dollars next year, but would rather have \$100 today, no problem. I would simply borrow \$100 at 10% interest, and pay back the \$110 I will get next year. .

\$100 today and \$110 one year from today have the same value (if the interest rate is 10%). They are the same economic entity.

This is what is meant by these present value formulas, (X / P) = (1 + R)

We can convert future dollars into their current dollar equivalents, and thus compare alternatives quite clearly. We can manipulate this equation to solve a number of different types of questions.

P = present amounts (today's dollars)
X = future amounts (future dollars)
R = nominal interest rate

P = X / (1+R) Gives us the present value of a future payment X, with an interest rate of R% per year. This answers the question: how much is X next year worth in terms of today's dollars.

X = P * (1+R) Gives us the future value of a present amount X, with an interest rate of R% per year. This answers the question: how much will I have if I put P in the bank for a year.

R = (X / P) – 1 Gives us the interest rate R, which will make us indifferent between P today and X next year.

Examples

Suppose R = 25%. What is the present value of a payment of \$250 to be received one year from today?
P = X / (1 + R) = \$250 / (1 + 0.25) = \$250 / 1.25 = \$200. That is \$250 to be received one year from now has the same value as \$200 today.

Suppose R = 15%. If you deposit \$40 in the bank for one year, how much will you have at the end of the year?

$X = P * (1 + R) = \$40 * (1 + 0.15) = \$40 * 1.15 = \$46$. That is \$40 today can be converted into \$46 next year, and thus they have the same value.

Suppose you are given the choice between receiving \$1000 one year from now and \$800 today. At what interest rate will you be indifferent between choosing these two options?

$R = (X / P) – 1 = (\$1000 / \$800) – 1 = 1.25 – 1 = 0.25$ (25%). That is, if the interest rate if 25%, you will be indifferent between having \$800 and \$1000.

<u>Extensions / Queries</u>

(1) What if the future payment is more than one year away?

$P = X / (1 + R)^n$ or $X = P * (1 + R)^n$ where n = number of years away

The logic is simple. Suppose it is the year 2001, and I want to know what a \$1 in 2003 is worth (two years away). Suppose I momentarily pretend its 2002. The dollar I will get in 2003 is now one year away. The present value of it is $\$1 / (1 + R)$ in terms of 2002 dollars. But now I know what its worth in 2002 dollars, and I can ask what its worth in 2001 dollars. I take $\$1 / (1 + R)$ (the value in 2002) and divide by (1+R). This gives me $P = A / (1 + R)^2$. Then we generalize.

Suppose R = 25%. What is the present value of a payment of \$250 to be received two years from today?

$P = X / (1 + R)^n = \$250 / (1 + 0.25)^2 = \$250 / 1.25^2 = \$250 / 1.56 = \$160$

Suppose R = 15%. If you deposit \$40 in the bank for three years, how much will you have at the end of three years?

$X = P * (1 + R)^n = \$40 * (1 + 0.15)^3 = \$40 * 1.15^3 = \$40 * 1.521 \approx \$61$

(2) What if there is more than one payment?

P (bundle) = P (payment1) + P(payment2) ....

In short, you can up the present values of multiple payments without any trouble.

If you were told that the present value of \$250 to be received in two years was \$200 when R = 25%, and the present value of \$125 to be received in one year was \$100 when R = 25%, then the present value of both payments together is simply \$200 + \$100 = \$300

(3) Does an infinite series of payments have a finite value?

Yes, this is called perpetuity (This is short for a perpetual annuity. An annuity (x) is the same amount paid in succession for some period of time. A perpetuity has payments forever).

If a constant payment a is received <u>one year from now</u>, and every year there after, then

$P = x / R$

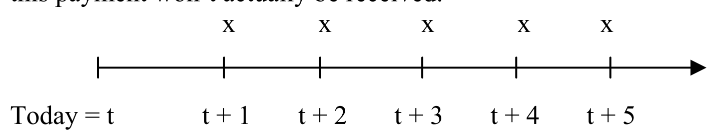
If the first payment is received today, we must add in the PV of the first payment, and thus

$P = x + (x / R)$ (see below for why this is the case)

If the first payment is received in the second year, we must subtract out the PV of the payment one year from today.

$P = (x / R) - x / (1 + R)$ (see below for why this is the case)

The general formula for a perpetuity adds up the present value of all of the payments shown below with "x"s, with the first payment occurring one year from today. If the first payment is received today, we must add in the present value of the payment occurring today, because that is not included in x / R. If the first payment is received two years from now, we must subtract out the present value of the payment occurring one year from today, because it is included in x / R, but this payment won't actually be received.



(4) Do I have to compare present values to make decisions?

No, you can use future values as well. It will make no difference. However, it seems most intuitive to calculate present values, as it express the future amounts in today's dollars, which are easier to understand than future dollars. So long as you compare both alternatives using the same dollars (compare Ps to Ps or compare Xs to Xs) you'll be fine.

(5) Are there any short cuts that make it easier to make some calculations?

Yes, the rule of 70 may be useful. It is just an approximation, but is sometimes useful.

Rule of 70

If something is growing at G % per year, the time it takes to double in magnitude is

$T = 70 / G$

We call T the doubling period
G is the growth rate

If your hair is growing at 7% per year, how long will it take to double in length?

$T = 70 / G = 70 / 7 = 10$ years

If your weight is doubling every 14 years, how fast is it growing?

$T = 70 / G$ $G = 70 / T$ $G = 70 / 14 = 5$ It is growing at 5% per year.

More Rule of 70

If you put P in the bank, and it doubles every N years, how much will you have at the end of T years?

$X = P * 2^{(N/T)}$

Where P = the present amount
N = the number of years
T = the doubling period

Put $100 in the bank at 3.5% per year for 40 years. How much do I have at the end?.

First, calculate the doubling period, t

Thus $T = 70 / G = 70 / 3.5 = 20$ ...it doubles every 20 years.

Then use the formula

$$X = X_0 * 2^{(N/T)} = \$100 * 2^{(40/20)} = \$100 * 2^{(2)} = \$100 * 4 = \$400$$

The rule of 70 is really just an approximation to $X = P * (1 + R)^T$, but easier on our calculators.

Here = $\$100 * (1.035)^{40} = \$100 * 3.959 = \$395.90$

This formula looks more complicated than it is. In the above example, we are just finding out that it doubles every 20 years. Thus, in 40 years, it will double twice. \$100 ⇒ \$200 ⇒ \$400.

<u>What types of questions will I be asked about?</u>

The questions you will see will usually give you two (or three) options to choose from.

1. Would you rather **pay** \$100 today or \$120 next year, given a nominal interest rate of 10%?

   The \$100 today has a present value (in terms of today's dollars) = \$100.

   \$120 next year has a present value (today's dollars) of \$120 / 1.10 = \$109

   Why? If you had \$109 dollars today, you could put it in a bank, earn 10% interest on it, and have \$120 next year. Thus, \$120 dollars next year is worth \$109 in terms of today's dollars.

   Given the choice between paying \$100 in today's dollars or something worth \$109 in today's dollars, which do you choose? The answer is clearly paying the \$100 today. You want to choose the payment with the smallest present value.

2. Suppose you have won the lottery, and the folks have given you two choices. **Collect** \$250 today or \$330 one year from now. Which do you choose given a nominal interest rate of 10%?

   The \$250 today has a present value of \$250.

   The present value of \$330 next year has a present value of \$330 / (1.10) = \$300.

   Why? If you had \$300 today, you could put it in the bank, earn 10% interest, and have \$330 in a year. You can turn \$300 today into \$330 next year. They have the same value.

   Given that you have won the lottery, which would you rather receive? The one with the higher present value. You should select the \$330 next year option.

3. Repeat #2, but instead of comparing the options in terms of this years dollars, use next years dollars. Should you get a different answer?

   The \$250 today has a future value $X = P * (1 + R) = \$250 * 1.1 = \$275$

   The \$330 next year has a future value of \$330.
   Thus, again, you would chose to collect the \$330 next year.

   The result is that it doesn't matter whether you compare in today's dollars or next year's dollars, so long as you're consistent (Rule #4). Present values tend to be more convenient.

	In today's dollars	In next years dollars
\$250 today	\$250	X = P * (1 + R) = \$250 * (1.1) = \$275
\$330 next year	P = X / (1 + R) = \$330 / 1.1 = \$300	\$330

Summary

The biggest point is that a dollar today is more valuable than a dollar tomorrow. You can put a dollar today in the bank at a nominal interest rate R, and next year you will have (1 + R) dollars. Thus, they have the same value, and you should be indifferent between these two choices.

Much like a Canadian \$5 bill and a US \$5 bill are not the same thing, a \$5 bill today and a \$5 bill tomorrow are two distinct goods. You know you need an exchange rate to convert the first two, but you also need an exchange rate between the second two. The nominal interest rate (really 1 + R) is that exchange rate between current dollars and future dollars.

The formula sheet

Future payment 1 year away: $P = X / (1 + R)$ $X = P * (1 + R)$ $R = (X / P) – 1$

Future payment n years away: $P = X / (1 + R)^n$ $X = P * (1 + R)^n$ xxxxx

Perpetuity beginning today $P = (x / R) + x$
Perpetuity beginning 1 yr. from today $P = (x / R)$
Perpetuity beginning 2 yr. from today $P = (x / R) – x / (1 + R)$

Time for something growing at G% per year to double: $T = 70 / G$

Let P, which doubles every T years, grow for N years: $X = P * 2^{(N/T)}$

Optional stuff for math jocks

The algebraically inclined will notice the following about the present value formulas:

- If you let n = 1 in the n year formula, you will get the one year formula. The one year formulas are just a simplified expression of the n year formulas.

- The three equations (for example in the one year case) are the same expression, solved for each of the arguments separately. It is not three equations, simply one equation solved for three different variables. (Thus the first 6 equations are really just one equation).

- The perpetuity formula is a result of a geometric series, where the "factor" is 1 / (1 + R). If you're interested, come by and I'll show you.

How good is the approximation in the rule of 70? For the curious, as the growth rate (G) gets larger, this approximation gets worse.

> In class, we let \$200 grow at 10% per year for 28 years. The doubling period was 7 years, thus, it doubled four times. Thus, the rule of 70's approximation suggested we would have \$3200. In actuality, we would have \$200 * 1.1^{28} = \$2884. Not super close, but in the neighborhood.
>
> Just for comparison sake, if we let the same \$200 grow at 2.33% per year for 30 years (roughly the same number of years), the doubling period would be 30 years. Thus, the \$200 would double once. The rule of 70's approximation would suggest we would have \$400. In actuality, we would have \$200 * 1.0233^{30} = \$399.13

What should I be reading?

Chapter 14 – The first part of this chapter gives you an intro to present value, you can read it. The second part is a portion about risk and insurance markets and diversification, and while interesting, we probably won't spend too much time on this. Also, the section on asset valuation would probably be interesting to many of you, but we'll leave this for the finance department.

Chapter 13 –

There's a portion on financial intermediaries that is interesting. Pay particular attention to how these intermediaries link savers and borrowers. It will come in handy more for the next lecture, as will the section on the supply and demand for loanable funds. I'll tell you again in a couple of lectures to read this chapter, but it woudn't hurt to do it now.