

Game Theory – a 30-minute primer

Game theory is the general theory of strategic behavior. It has applications in Biology, Political Sciences, Psychology, the Military, and of course Economics. Essentially, we set up “games” where we can look at the behavior of individuals. Most games involve strategic interactions among players. We are interested in looking at how a player’s actions affect them and affect the actions of the other players.

We call participants in the games **players**.

The **rules of the game** say who can do what and at what times.

A player’s **strategy** is a plan for actions in each people situation in the game.

A player’s **payoff** (outcome) is the amount that the player wins or losses in a particular game situation.

So, a very simple example would be the game of rock, scissors, paper. There are two players. The rules are simple. Each player simultaneously picks one of the three strategies, rock, scissors, or paper. Depending on the strategy that each person chooses, one of the players wins, or there is a draw.

This is an example of a game where both players choose their strategy simultaneously. Thus, neither player knows what strategy or choices the other player has made. Later, if you learn more game theory, you will talk about games where players take turns, have multiple moves, etc. It gets more complicated, but it gets more interesting and informative as well.

A player’s **best response** (best strategy) is the strategy that maximizes that player’s payoff, given the strategies of other players.

A player has a **dominant strategy** if that player has a best strategy that does not depend on what other players do.

Suppose I know that my opponent is going to play paper. My best strategy is to play scissors, as this will result in me winning. Scissors are my best response.

However, if I knew my opponent was going to play rock, my best response would be to play paper. Paper is my best response.

Thus, for this game, there is no dominant strategy, as my best response changes depending on the strategy that my opponent chooses.

An example of a game with a dominant strategy follows: a little known variation of Rock, Scissors, Paper is to allow choosing dynamite. At least at my house when I was little, dynamite beat rock, scissors, and paper. Thus, regardless of what your opponent chose, it was optimal to choose dynamite. Thus, dynamite was the best response to any strategy that my opponent used, thus it was a dominant strategy. Having a dominant strategy makes it easy to figure out what is going to happen, but doesn’t make for a very fun game to play (everyone plays dynamite). Which of course leads to atom bomb, etc...

A **Nash equilibrium** is a situation in which every player makes his best response. It occurs when each player’s strategy is optimal, given the strategy of other players.

As it turns out, in any game where both players have a dominant strategy, the Nash equilibrium is simply the result of each player using their dominant strategy.

Example, Bonnie and Clyde – the prisoner’s dilemma

Bonnie and Clyde are out on a crime spree. They are caught for a minor crime, but the police chief wants to pin them for the more severe crime that they are suspects for (and have committed). However, they need

the testimony of one of the suspects to convict the other. The person who rats out the other will get a break. Strategies and payoffs for each are listed below. Of course, each of Bonnie and Clyde want to spend the least amount of time possible in prison.

		Clyde	
		Confess	Don't Confess
Bonnie	Confess	5 yrs for C, 5 yrs for B	10 yrs for C, 1 yr for B
	Don't Confess	1 yr for C, 10 yrs for B	2 yrs for C, 2 yrs for B

Assume, momentarily, that Bonnie was to confess. What is Clyde's best response? Well, if Clyde confesses, he gets 5 years, but if he doesn't confess, he gets 10 years. Thus, Clyde will confess.

Suppose, now, instead, Bonnie chose not to confess. What is Clyde's best response? If Clyde confesses, he gets 1 year, but if he doesn't confess, he gets 2 years. Thus, Clyde will confess.

Clyde's dominant strategy is to confess. No matter what strategy Bonnie chooses, he will confess.

If you repeat the logic above, you will also find that Bonnie's dominant strategy is also to confess. Bonnie will also confess.

Thus, the Nash equilibrium is for both to confess. How can we check it is really a Nash equilibrium? This strategy has to be the best response (for both players) to the strategy the other is choosing. That is, given Bonnie is confessing, is it Clyde's best response to confess? Yes. Given that Clyde is confessing, is it Bonnie's best response to confess? Yes. Thus, indeed it is a Nash equilibrium. Both will confess. This will be the outcome of the game. That is, both Bonnie and Clyde will confess.

Now, for the interesting part?

We figured out the Nash equilibrium strategy was for both to confess, resulting in 5 years for each. However, if they had just got together and agreed not to confess (cooperated) they could have gotten out with only 2 years each.

We have a situation where each of the individuals has an individual incentive to choose a course of action that ends up being harmful for "the group" as a whole.

Even though it would be better for both if they would agree to not confess, if one of the players would agree not to confess, the other would have the incentive to confess. The agreement (not to confess) is difficult to enforce.

Ok, the numbers are unreasonable? This doesn't sound like anything that would happen in the real world, does it? I disagree. We have implicitly talked about several situations that fall under the realm of the prisoner's dilemma. We call this sort of set up the **prisoner's dilemma**, as the original example of Bonnie and Clyde. There are many situations where the payoffs look similar to this situation.

Some pseudo real life examples of the prisoner's dilemma

How about a cartel between Coke and Pepsi, with payoffs representing profits. They agree to restrict output. But, might each have an incentive to cheat.

		Pepsi	
		Cheat	Don't Cheat
Coke	Cheat	\$3 million each	C gets \$8 million, P gets \$2 million
	Don't Cheat	C \$2 million, P gets \$ 8 million	\$5 million each

Same as before. Both will have dominant strategies to cheat. The cartel will break down, even though they could have earned a bunch of profits if they stuck to the agreement.

Or how about a trade war between the US and Japan?

Maybe the US could stick a tariff on a good in which the US was a large nation, making the US better off. Or Japan could do the same.

		Japan	
		Tariffs	No Tariffs
US	Tariffs	\$5 GFT each	\$9 for US, \$4 for Japan
	No Tariffs	\$4 for US, \$9 for Japan	\$8 each

Same as before. We get a trade war with lots of tariffs, even though we know GFT are maximized with undistorted trade.

What about contributions to a public good? Everyone has the incentive (dominant strategy) to free ride, but this results in no public goods (the bridge doesn't get built). But if everyone had contributed...

The point is that the prisoner's dilemma is pervasive, and game theory can be used to explain a lot of the stuff we have already studied.

Be sure that you can see why we get the equilibrium we do. Best responses, dominant strategies, etc....

What's the bigger picture?

So far, the point is another illustration of one of things I have been trying to stick into your head throughout the semester. When we see outcomes that are harmful to society as a whole, the reason is that some person or group has an incentive to do something else. Economics is about incentives. In game theory, we can explicitly model people's incentives. Can we alter the incentives of those people to get the outcome that makes society better off? Can the prisoner's dilemma be solved?

(Aside. When I say society better off in this context, I am talking about society as the group of players involved with this game.)

Can the prisoner's dilemma be solved? Answer yes. Think back the original situation, with Bonnie and Clyde. If it is a one-time deal, where the criminals don't know (trust) each other, we might expect that criminals to each confess. However, if they are long time criminals and married, we might find it more likely that they cooperate.

The problem, just like it was in the cartel, is that you have an agreement that is difficult to enforce. If you could enforce the agreement (agree to cooperate) you could do better. In general, if we allow the same players to play the game multiple times, we usually can get cooperation, for a little while. However, if there is ever an end to the association, it will break down. Your textbook, if you are interested, talks about strategies that will get you this result. Tit for tat for example...

Or suppose that Bonnie can make some additional threat to Clyde. Bonnie tells Clyde that she can have him wacked even from inside prison. (Really, this would be a change in the payoffs, so we'd have a structure of the game that wouldn't be a prisoner's dilemma properly.) This is called credible commitments. As an example, in the Pepsi / Coke cartel, we could have Coke (and Pepsi) agree to shut down a few bottling companies. This would make Coke commit to not expanding output (the low price).

Read the book for more information if you like. Really, game theory has a lot of interesting applications, of course, that I cannot tell you about in a half an hour on the last day of class. Some games have multiple Nash equilibria. Some games are sequential games; players take turns (TIC, TAC, TOE). Some games allow commitments to strategies (Coke and Pepsi). Some games are played multiple times (Bonnie and

Clyde go on several bank heists). This changes things around. Interestingly enough, the game theory was originally developed as a tool by the military to consider military strategies (nuclear stuff).

As one more example, consider a prisoner's dilemma set up as follows. There is a duopoly - that is only two producers of the good. They are considering which prices to charge.

		Jack	
		High Price	Low Price
Jill	High Price	Jack \$7500, Jill \$7500	Jack \$9500, Jill \$1000
	Low Price	Jack \$1000, Jill \$9500	Jack \$5000, Jill \$5000

If both charge high prices, they earn high profits. If one of the players charges a low price (while the other chooses a high price) they get the bulk of the business and earn large profits (while the other earns less). Of course, if they both choose a low price (expand output) profits decrease.

So, each has a dominant strategy of charging the low price (confirm this). The Nash equilibrium will be Low Price, Low Price. This is the same as before. Can we solve the dilemma, or at least find a way around it?

Consider a Guaranteed Price Matching Agreement

Suppose Jill places an Ad in the newspaper that guarantees that she will match any competitor's price. Suppose for a second that Jill charges the high price. Does Jack have an incentive to charge the low price and try to steal away all of the business? The answer is no, since if Jack charges the low price, Jill will match the low price. They would both get \$5000 of profits. Jack will no longer have any incentive to charge the low price. If Jack also makes the same advertisement, Jill would no longer have any incentive to charge the low price. Thus, the guaranteed price matching agreement is a sort of form of enforcing a cartel. Weird, eh?