Elasticity Formulas

Conceptually, elasticity is a measure of responsiveness. Algebraically, it is the ratio of two percentage changes. When you think elasticity, you should think "responsiveness".

When we write "% Δ X" - this means the percentage change in X

Elasticity of Demand – (the own price elasticity of demand) – this measures the response of quantity demanded to a change in this good's own price.

It can also be interpreted as follows. If the elasticity of demand is –2, then a 1% increase in price will lead to a 2% decrease in quantity demanded. Likewise, if the elasticity of demand is –4, a 1% decrease in price will lead to a 4% increase in quantity demanded.

$E_D = \% \Delta Q_d / \% \Delta P$

By the 1st Law of Demand, this must be negative.

If $\% \Delta P > 0$, then $\% \Delta Q_d < 0$ $\Rightarrow$ $E_D < 0$
If $\% \Delta P < 0$, then $\% \Delta Q_d > 0$ $\Rightarrow$ $E_D < 0$

To keep things from being less confusing, we will often talk about the absolute value of the elasticity of demand (remove the negative sign).

$| E_D | > 0$.

There will be three relevant ranges of the elasticity of demand.

$-\infty < E_D < -1$, elastic	Alternatively,	$1 < \| E_D \| < \infty$, elastic
$E_D = -1$, unit elastic		$\| E_D \| = 1$, unit elastic
$-1 < E_D < 0$, inelastic		$0 < \| E_D \| < 1$, inelastic

Also,

$E_D = -\infty$, perfectly elastic		$\| E_D \| = \infty$, perfectly elastic
$E_D = 0$, perfect inelastic		$\| E_D \| = 0$, perfectly inelastic

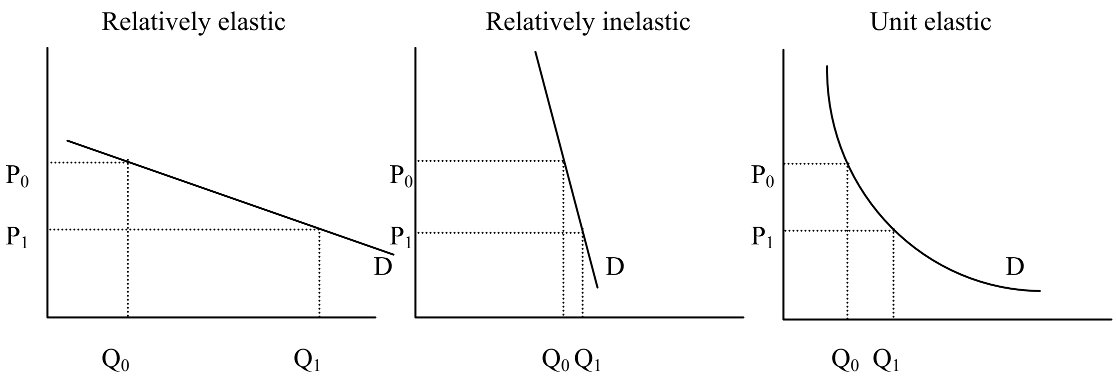
Suppose there is a 20% increase in price that leads to a 40% reduction in quantity demanded.
In this case, $\% \Delta P = 20\%$, $\% \Delta Q_d = -40\%$

$E_D = \% \Delta Q_d / \% \Delta P = -40\% / 20\% = -2$ This would be an elastic demand curve ($|E_D| = 2$).

Suppose a 60% increase in price leads to a 10% reduction in quantity demanded.
In this case, $\% \Delta P = 60\%$, $\% \Delta Q_d = -10\%$

$E_D = \% \Delta Q_d / \% \Delta P = -10\% / 60\% = -0.16$ This would be an inelastic demand curve ($|E_D| = 0.16$).

Below, I have sketched a relatively elastic, inelastic and unit elastic curve. I am cheating a bit. Along any linear demand curve is the whole range of elasticities. I think this is slightly beyond the scope of the knowledge we will need for our class. See the book or the end of notes if you care (I can't blame you for not caring), but if not, just trust me that I am cheating a little bit and don't worry too much about this. Elasticities are <u>not</u> the same thing as the slope of the demand curve.

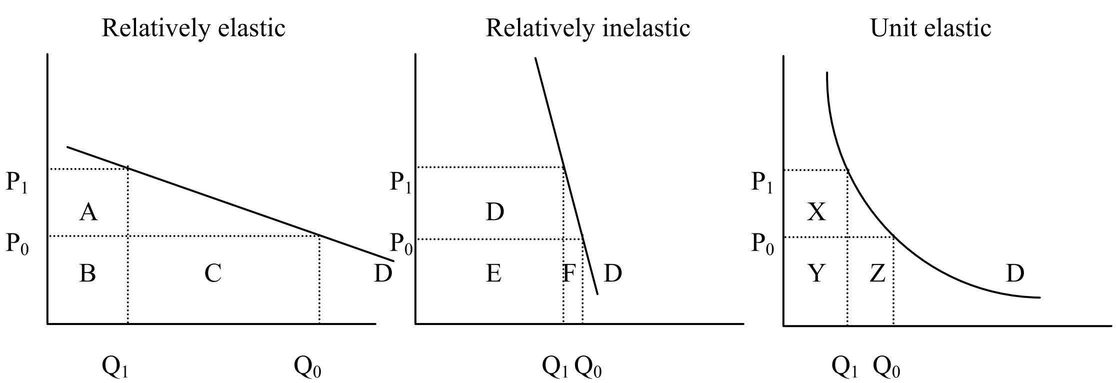


Notice that for the relatively elastic curve, we see a large change in quantity demanded for a given % change in price. That is, demand is very responsive or elastic. If you'd rather, the % Δ Q_d is larger in magnitude than the % Δ P.

However, for the relatively inelastic curve, we see only a modest change in quantity demanded for the given % change in price. We would say that this demand curve is unresponsive, or inelastic. If you'd rather, the % Δ Q_d is smaller in magnitude than the % Δ P.

<u>What happens to the total expenditures (spending) on a good as the price changes?</u>

We can use the elasticity concept to determine what happens to total expenditures (total spending) as we move along a demand curve. Algebraically, total expenditures is equal to price * quantity. Graphically, it is the rectangle formed by P_0 and Q_0 (or P_1 and Q_1). This is a little tricky, because price and quantity move in the opposite direction. An increase in price say, leads to a decrease in quantity demanded. What happens to total expenditures depends on which is bigger, which is precisely what an elasticity can tell us…



We will consider all three cases for a price increase. You can do the decreases if you like. (In class we did inelastic demand curves first, and elastic demand curves second).

Case 1: Elastic Demand
$\text{Expenditures}_0 = P_0 * Q_0$ = boxes B and C
$\text{Expenditures}_1 = P_1 * Q_1$ = boxes B and A

Since box B is involved in both, to decide which is bigger, we must only compare the sizes of box A vs. Box C. As you can see, box C is larger. Thus, expenditures were larger originally, and thus as a result of the price increase expenditures have fallen.

Result: For an elastic demand curve, $\uparrow P \Rightarrow \downarrow$ Total expenditures
For an elastic demand curve, $\downarrow P \Rightarrow \uparrow$ Total expenditures

Case 2: Inelastic Demand
$Expenditures_0 = P_0 * Q_0$ = boxes E and F
$Expenditures_1 = P_1 * Q_1$ = boxes E and D

Since box E is involved in both, to decide which is bigger, we must only compare the sizes of box D vs. Box F. As you can see, box D is larger. Thus expenditures have increased.

Result: For an inelastic demand curve, $\uparrow P \Rightarrow \uparrow$ Total expenditures
For an inelastic demand curve, $\downarrow P \Rightarrow \downarrow$ Total expenditures

Case 3: Unit elastic Demand
$Expenditures_0 = P_0 * Q_0$ = boxes Y and Z
$Expenditures_1 = P_1 * Q_1$ = boxes Y and X

Since box Y is involved in both, to decide which is bigger, we must only compare the sizes of box X vs. Box Z. If drawn properly (a rectangular hyperbola), box X and box Z will be the same size. Expenditures have not changed as a result of the price increase. In fact, for a unit elastic demand curve, the change in quantity will be offset exactly by an equal sized change in price. The percentage change in price will be the same magnitude as the percentage change in quantity.

Result: For a unit elastic demand curve, $\uparrow P \Rightarrow$ no change in expenditures
For a unit elastic demand curve, $\downarrow P \Rightarrow$ no change in expenditures

What sorts of things tend to be inelastically demanded? We'll see more later, but for now, think of things that have few good substitutes. Examples here included cigarettes, gasoline, and brain surgery. Goods that have a lot of good substitutes will tend to be elastic, such as coke, pretzels, and Hanes T-shirts. More next time.

Elasticity of Supply – (the own price elasticity of supply) – this measures the response of quantity supplied to a change in this good's own price.

It can also be interpreted as follows. If the elasticity of supply is 2, then a 1% increase in price will lead to a 2% increase in quantity supplied. Likewise, if the elasticity of supply is 4, a 1% decrease in price will lead to a 4% decrease in quantity demanded.

$E_S = \% \Delta Q_S / \% \Delta P$

By the 1st Law of Supply, this must be positive.

If $\% \Delta P > 0$, then $\% \Delta Q_s > 0 \quad \Rightarrow \quad E_S > 0$
If $\% \Delta P < 0$, then $\% \Delta Q_s < 0 \quad \Rightarrow \quad E_S > 0$

There will be three relevant ranges of the elasticity of supply.

$0 < E_S < 1$, inelastic
$E_S = 1$, unit elastic
$1 < E_S < \infty$, elastic

Also,
$E_S = \infty$, perfectly elastic
$E_S = 0$, perfectly inelastic

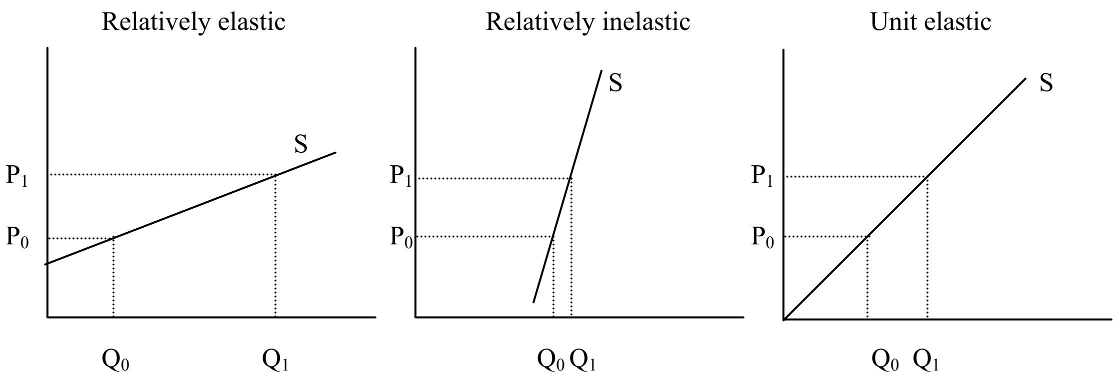
Suppose there is a 30% increase in price that leads to a 20% increase in quantity supplied.
In this case, $\% \Delta P = 30\%$, $\% \Delta Q_s = 20\%$

$E_S = \% \Delta Q_s / \% \Delta P = 20\% / 30\% = 0.67$ This would be an inelastic supply curve.

Suppose a 40% decrease in price leads to a 60% reduction in quantity supplied.
In this case, $\% \Delta P = -40\%$, $\% \Delta Q_s = -60\%$

$E_S = \% \Delta Q_s / \% \Delta P = -60\% / -40\% = 1.5$ This would be an elastic supply curve

Again, the save caveat about cheating applies again. However, pictures of relatively inelastic and elastic supply curves are drawn below. Elasticities of supply are <u>not</u> the same thing as the slope of supply curves.



Notice that for the relatively elastic curve, we see a large change in quantity supplied for a given % change in price. That is, the supply curve is very responsive or elastic.

However, for the relatively inelastic curve, we see only a modest change in quantity supplied for the given % change in price. We would say that this supply curve is unresponsive, or inelastic.

For the unit elastic supply curve, the change in quantity supplied and the change in price are of the same size.

Regardless of the elasticity of supply, total revenue (price * quantity) always moves in the same direction as price (and quantity supplied).

An in increase in price will lead to an increase in quantity supplied. Thus, revenue will increase.
A decrease in price will lead to a decrease in quantity supplied, thus, revenue will decrease. You can shade in the boxes above as we did with demand, but you will find it is pretty trivial.

Note: When I am talking about elasticities of demand, I am calling P * Q = total expenditures. When I am talking about supply elasticities, I call P * Q = total revenue. Conceptually, they are no different. I am just doing this to try to keep it clear which type of elasticity we are talking about.

<u>**Summary, thus far**</u>

Demand

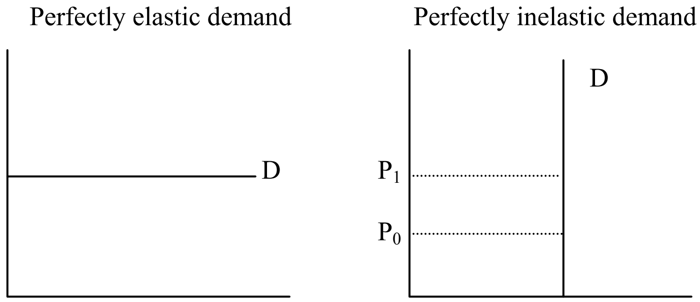
	Elastic	Inelastic	Unit Elastic
P ↑	TE ↓	TE ↑	no Δ
P ↓	TE ↑	TE ↓	no Δ

Supply

	Elastic	Inelastic	Unit Elastic
P ↑	TR ↑	TR ↑	TR ↑
P ↓	TR ↓	TR ↓	TR ↓

Extremes of elasticities

In the extreme, a demand curve can be perfectly elastic ($|E_D| = \infty$), or perfectly inelastic ($|E_D| = 0$).



The perfectly inelastic case (right) is the more intuitive. In response to a change in price, there is no change in quantity demanded. $\% \Delta Q_d = 0$. There is no response at all to the change in price. Hence the term, perfectly inelastic. If you look at the formula, it turns out the value of $|E_D| = 0$.

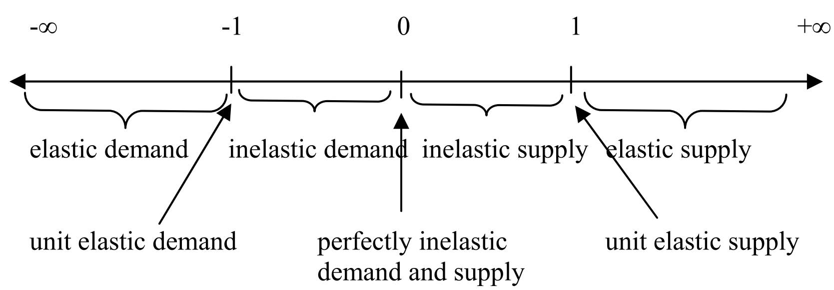
The perfectly elastic case is a bit tricky. Here, if the price changes, there is an infinite response of change in quantity demanded. If the price raises by a fraction of the penny, quantity demanded goes to zero. This is extremely responsive. $|E_D| = \infty$.

It is also the case, that in the extreme, supply curves can be perfectly elastic ($E_S = \infty$), or perfectly inelastic ($E_S = 0$). The pictures are the same, you only need to re-label the curves with a S instead of D.

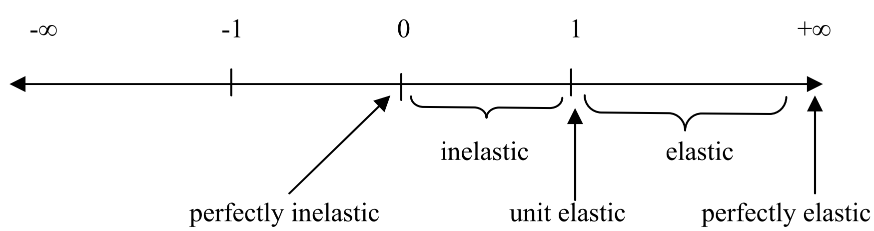
In the perfectly inelastic case, there is no change in quantity supplied in response to the increase or decrease in price. $E_S = 0$.

In the perfectly elastic case, if the price rises a fraction of a penny, the quantity supplied goes to zero. $E_S = \infty$.

Numberline



If you flip over the negative half of the number line onto the positive half, (that is we are now considering the absolute value of the elasticity of demand) you can see, **for both S and D**,



Or if you'd prefer this is symbol form…

Perfectly inelastic	E_S or $\mid E_D \mid = 0$
Inelastic	$0 < E_S$ or $\mid E_D \mid < 1$
Unit elastic	E_S or $\mid E_D \mid = 1$
Elastic	$1 < E_S$ or $\mid E_D \mid < \infty$
Perfectly elastic	E_S or $\mid E_D \mid = \infty$

Are there any more interesting elasticities to calculate? Maybe not, but there are a few more…

Unlike the elasticity of demand and the elasticity of supply (which refer to movements along a given S or D curve), the following elasticities of demand deal with changing ceteris paribus conditions (income, price of related goods) and thus refer to shifts of the demand curve.

Income elasticity of Demand

$$E_I = \frac{\%\ \Delta\ \text{Demand for good X}}{\%\ \Delta\ \text{Income of consumers}}$$

If $E_I > 0$, then good X is a normal good.
If $E_I < 0$, then good X is an inferior good.

If a 5% increase in the income of consumers leads to a 15% increase in the demand for good Z

$E_I = 15\% / 5\% = 3$ The good is normal.

If a 10% decrease in the income of consumers leads to a 5% increase in the demand for good Z

$E_I = 5\% / -10\% = -0.5$ The good is inferior.

We talked about this a great deal already. Check your notes on ceteris paribus conditions for more. Whether a good is a normal good or not is an empirical issue. That is, before I can decide if lunchboxes are a normal good, I should go out and do this calculation. In other words, I can't look at a good and decide it is a normal good until I do this calculation. It is because $E_I > 0$ that I say lunchboxes are a normal good.

Cross Price Elasticity of Demand

$$E_{XY} = \frac{\%\ \Delta\ \text{Demand for good X}}{\%\ \Delta\ \text{Price of good Y}}$$

If $E_{XY} > 0$, then goods X and Y are substitutes.
If $E_{XY} < 0$, then goods X and Y are compliments.

If a 20% increase in the price of good Y leads to a 15% increase in the demand for good X

$E_{XY} = 15\% / 20\% = 0.75$ The goods are substitutes.

If a 10% decrease in the price of good Y leads to a 5% increase in the demand for good X

$E_I = 5\% / -10\% = -0.5$ The goods are compliments.

We talked about this a great deal already. Check your notes on ceteris paribus conditions for more. Again, I don't know a priori if two goods are substitutes or compliments, I must do the calculations above.

You really want to know why an elasticity of demand is not the same thing as the slop of a demand curve?

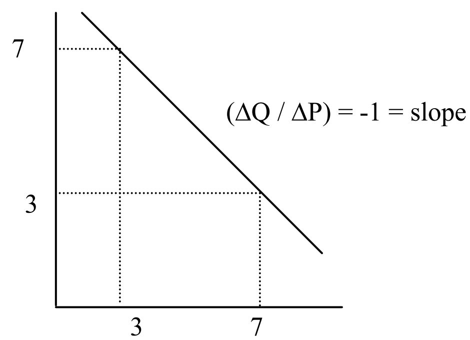
You can read the book if you want or check this out for the mathematically inclined.

$E_D = \% \Delta Q_d / \% \Delta P$

But since, a percentage change is just $(Q_1 - Q_0) / Q_0$, and letting $\Delta Q = Q_1 - Q_0$
The percentage change is just $\Delta Q / Q_0$.

Thus, $E_D = (\Delta Q / Q_0) / (\Delta P / P_0)$
$= (\Delta Q / \Delta P) * (P_0 / Q_0)$ from rearranging the above equation

But, $(\Delta Q / \Delta P)$ is the slope of the demand curve, and thus, if we draw a linear curve, is held constant. However, as we move along a demand curve, P_0 / Q_0 changes.



Thus, $E_D = (\Delta Q / \Delta P) * (P_0 / Q_0) = (-1) * (7 / 3) = -2.33$ when measured at the point on the upper portion of the demand curve.

While, $E_D = (\Delta Q / \Delta P) * (P_0 / Q_0) = (-1) * (3 / 7) = -0.43$ when measured at the point on the lower portion of the demand curve.

As we move along a linear demand curve, you get elasticities that range all the way from infinity (near the vertical axis) to zero (near the horizontal axis). But, if you pick a certain price, and calculate the elasticities for two demand curves, you'll find the demand curve with the steeper slope (relatively inelastic) will be more inelastic than the other demand curve. Again, don't worry about this a great deal.