Did we go wrong in class? – The easy way that everyone should understand

In class, we calculated MC by looking at the change in TC for each additional one unit. This would be the correct thing to do if only whole units of goods could be produced (think dining room tables). If you do some math (see below) it turns out you'd really have the largest profit level if you produced 2.5 units of the good.

In this case, since we can't really produce 2.5 tables, we are stuck with choosing between 2 and 3 units, and our decision rule says to produce 3 (the quantity where P = MC). In fact, as pointed out in class, you would be indifferent between producing the second unit and the third unit. You'd be willing to let someone decide whether or not you should produce the third unit. (Recollect that we assumed P = $5)

Output	TC	MC	TR	Profit
0	$0	----	$0	$0
1	$1	$1	$5	$4
2	$4	$3	$10	$6
3	$9	$5	$15	$6
4	$16	$7	$20	$4
5	$25	$9	$25	$0

In the interest of keeping our rule consistent, let's produce the third unit. This will keep our decision rule consistent, and keep us from having to say our decision rule is something as ridiculous sounding as this:

> Firms find a rate of output where marginal cost is just equal to market price, and then produce one less unit than that (and it wouldn't be right all the time either…)

That just sounds bad. So let's produce the extra unit, and stick with the decision rule as presented in lecture.

> Firms choose a rate of output where marginal cost is just equal to the market price.

Seriously, don't let this bother you. Don't forget at the very most, we are talking about half a unit of the good here. This isn't worth losing sleep over, but for those gluttons for punishment, read on…

More details – not too bad even if you forgot most of your calculus?

The problem that we are having is that we are taking a continuous relationship and making it discrete.

If we were trying to come up with a mathematically elegant theory, it would be easiest to assume that we had continuous functions. That is, you can produce 1 unit of the good, or 1.27 units of the good, or 1.54 units of the good, etc…. Goods are perfectly divisible (think of something like gallons of milk).

If so, and we allow total cost to be a continuous function, let's denote it TC (q). In the example in class we had, this would have been $TC = Q^2$.

The next question is what is MC. Since we can produce fractions of a unit, it doesn't really make sense to talk about the MC of producing one unit. Instead we could talk about the cost of producing half a unit, or a third of a unit, 0.01 of a unit, or even smaller. So in fact, marginal cost is the instantaneous rate of change of TC for an infinitely small increase in output. If you didn't fall asleep in calculus class (I did), this is really the first derivative of TC.

In this case, MC = 2Q.

Thus, using our decision rule, and choosing the rate of output where P = MC = $5 boils down to choosing a rate of output where Q = 2.5. This indeed turns out to gives us the maximum profit level, as illustrated below.

Output	TC	TR	Profit	MC***
0	$0	$0	$0	----
1	$1	$5	$4	$2
2	$4	$10	$6	$4
2.25	$5.0625	$11.25	$6.1875	$4.5
2.4	$5.76	$12	$6.24	$4.8
2.5	$6.25	$12.5	$6.25	$5
2.6	$6.76	$13	$6.24	$5.2
2.75	$7.5625	$13.75	$6.1875	$5.5
3	$9	$15	$6	$6
4	$16	$20	$4	$8
5	$25	$25	$0	$10

Notice that we make a little bit of profit producing the units between 2 and 2.25, between 2.25 and 2.4, and finally between 2.4 and 2.5. That is, we are making money right up until the very spot that P = MC. Also notice, that if we produce any more than 2.5 units, we begin losing profits.

So, if the functions were continuous, we'd still get maximum profits by choosing the rate of output where P = MC. That is, with continuous function, our decision rule is correct.

<u>More for the calculus inclined</u>

If you look at the example above, the other problem is that we are not quite calculating marginal cost correctly. Ideally, it would be the first derivative, and thus we know the correct figures are the ones given below in the MC' column (see below).

What we are calling MC is really the average rate of change of TC from Q to Q + 1 units, which is not the true derivative. Again, with a continuous function and the magic of calculus, we can figure out the instantaneous rate of change. If you look at the MC' column we see that you should choose Q = 2.5, and our decision rule is correct.

Output	TC	MC	TR	Profit	MC'
0	$0	----	$0	$0	----
1	$1	$1	$5	$4	$2
2	$4	$3	$10	$6	$4
2.5	$6.25		$12.5	$6.25	$5
3	$9	$5	$15	$6	$6
4	$16	$7	$20	$4	$8
5	$25	$9	$25	$0	$10

<u>One last thing – can I salvage Q=3?</u>

If you're still reading this (fluffy dragon marshmallow breath), there's one last possibility.

I said above that we were taking TC as given, and we weren't calculating MC correctly. What if in fact it was the opposite? What if it was the MC figures that were correct (1, 3, 5, 7, 9), and instead the TC figures were all wrong?

Since MC is the derivative of TC, to get TC from MC we can integrate the MC curve.

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Thus, TC (q) = ∫ MC (q) dq

In this case, we can say MC = -1 + 2Q, and integrate, arriving at TC = Q^2 – Q.

If so, we have a different unfortunate problem, but as you see, the profit maximizing output is actually 3 units of output. Once again, profit is maximized by choosing the rate of output such that P = MC. This occurs at Q = 3.

Output	TC	TR	Profit	MC
0	$0	$0	$0	----
1	$0	$5	$5	$1
2	$2	$10	$8	$3
3	$6	$15	$9	$5
4	$12	$20	$8	$7
5	$20	$25	$5	$9

As you can see, this is super screwy.

For those of you who made it this far

Don't let it bother you. Accept that we are making a continuous relationship discrete, and always produce at the rate of output where P = MC.