HUMAN CAPITAL PRODUCTION WITH PARENTAL TIME INVESTMENT IN EARLY CHILDHOOD

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This paper considers parental time investment in early childhood as an education input and investigates its relationships with other inputs in their contribution to human capital. I develop a 12-period overlapping generations model where human capital formation is a multistage process. The model is calibrated to the US economy so that the generated data matches patterns in parental education spending and child care time. The estimation results show that time input is complementary to education expenditure. I apply the model by implementing three early education policies. The first two involve more government spending and the third is paid parental leave. The policy experiments show that adopting paid parental leave is the most costly means of increasing human capital. An education subsidy is more effective than a direct increase in government spending at promoting human capital; however, its impact on earnings inequality and persistence is smaller.

Keywords: Government Education Expenditures, Life-Cycle Model, Early Childhood Education, Paid Parental Leave

1. INTRODUCTION

There is compelling evidence from economics and sociology that early education plays a unique role in human capital accumulation. Quality learning in early years has long-term effects on educational attainment and labor market outcomes. Missed opportunities to invest in young children cannot be easily compensated for by later investments. Economists capture this feature by modeling human capital formation in a hierarchical fashion, where early and late education interact in producing human capital. Most studies consider expenditures by parents and the government as the only education input. However, many parents invest large amounts of time caring for their young children and teaching them words, numbers, and manners. Such time investments can be very costly. Haveman and Wolfe...
show that the monetary cost of maternal care time accounts for 18% of total parental investment in early education.

This parental time investment is valuable to early development. Landry et al. (2003) argue that parent–child interactions help children understand that others respond to their interests and needs. With that understanding, children are more likely to explore and learn in an active way. An active learning experience in early years provides a good foundation for later learning. Belsky et al. (2007) show that nonparental care is associated with children’s vocabulary and behavior problems. This result implies that parents may have positive influences on children’s noncognitive skills. Grusec and Goodnow (1994) point out that experience with parents gives children a sense of security and teaches them effective ways to communicate.

Given the significant size and important role of parental time, I investigate how time investment as an education input contributes to human capital. As in Caucutt and Lochner (2011), human capital development is modeled as a multistage process. Children receive private and public education expenditures throughout childhood, while they receive time investment from parents only in early childhood. Therefore, parents of young children choose between work and child care with their time. Parents with older children provide one unit of inelastic labor supply. I choose this specification for two reasons. First, empirical evidence demonstrates that the presence of preschool children has a substantially larger impact on parental work time than the presence of older children. Second, parental involvement in early years is found to be most influential to children’s learning.

My focus is on the estimation of a human capital production process embedded in a dynamic life-cycle model. The calibrated model is successful in reproducing intergenerational persistence of earnings, cross-sectional variance in earnings, and several parental education investment trends in the United States. The key interest is the role of time input in its interaction with goods input when generating education outcome. Parental time can be substituted by education expenditures to some degree. For example, parents or the government can spend money on day care, which may crowd out maternal care time. However, some empirical work implies that this substitutability is low. Jonathan et al. (2008) show that parents with higher degrees tend to spend more time with children, particularly on education-related activities. Bernal and Keane (2011) find a negative impact of maternal participation in the labor market on child outcomes. My calibration result also infers a low substitutability between goods investment and parental time. Nordblom (2003) is inclined to the same assumption and argues that parents provide children with complementary skills through informal education at home.

I then perform a set of experiments on early education policy. This exercise illustrates how the model can be used to compare spending policies with a parental leave policy. Currently, the US government spends 0.34% of gross domestic product (GDP) on early education. I simply double this funding level in all experiments. In the first experiment, the government allocates increased resources to public spending directly. In the second, it subsidizes private spending. In my third experiment, I introduce paid parental leave to the economy. Both the subsidy rate
in the second experiment and the wage replacement rate in the third experiment are chosen at a level such that the policy is funded by an additional 0.34% of GDP. Therefore, my analysis provides comparable macroeconomic effects across alternative policies.

All the policies improve welfare because they partially correct a market failure in human capital accumulation. Parents in my model cannot borrow from the future earnings of their children. Given the altruism parameter chosen through calibration, people underinvest in their offspring, leading to a lower level of human capital in the steady state. Education polices help to remedy this distortion. Between the two policies targeting education expenditures, the subsidy is more effective at promoting human capital than direct government spending. This result is due to the crowding out of government spending, which weakens the policy effect. The paid leave program has the smallest impact on human capital among all the implemented policies. While it results in the largest increase in time investment, the wage compensation is spread over consumption and education spending, leading to a small increment in goods investment. This result is in line with Del Boca et al. (2014). They find that cash transfers generate little improvement in child quality.

My work is related to Blankenau and Youderian (2015). Both papers model early education inputs as relatively more productive complements to later inputs. Compared to their work, this paper has a similar model specification but introduces parental time investment. This feature results in a drastically different production function of early human capital and provides an environment for discussion on parental leave policy. While Blankenau and Youderian (2015) focus on the relationship between government expenditure and earnings persistence, my paper evaluates and compares spending polices with a parental leave policy. Such analysis requires an understanding of how parental time contributes to human capital.

Among a recent literature that models parental time as an education input, one set of papers use their theoretical model to explain certain empirical findings. Schoellman (2016) uses refugee data and shows that the arrival time of refugees does not impact their adult outcomes. His model provides an explanation that parental time, rather than country environment, is important to human capital. Intrigued by the correlation between educational attainment and parental income, Nordblom (2003) builds a model that reconciles two main arguments on its causes. She argues that parental investment is complementary to public spending and more public spending is more beneficial to children with rich parents.

Another set of papers are quantitative like mine. A common exercise in these papers is to quantitatively evaluate alternative policies. For example, Morchio (2013) uses his model to study the impact of a child allowances program. Del Boca et al. (2014) focus on a monetary transfer to parents. Both papers and others impose a certain degree of substitutability between time and goods input. My study departs from this work by estimating a general constant elasticity of substitution (CES) case, allowing for the substitutability to change. With a proper understanding of the role of parental time, I proceed with spending and parental leave policies. Zhu and Vural (2013) is an exception that also estimates and
finds complementarity between inputs. My model differs from theirs considerably with an embedded life-cycle optimization problem. This feature allows me to count for and estimate the interaction among education inputs across periods. The estimation of my model in such a rich economic environment better facilitates the policy analysis.

2. MODEL

I develop an overlapping generations model where time is discrete with an infinite horizon. The economy is populated by a continuum of agents who live for 12 periods and each period lasts six years. In the first three periods of life, agents are in early, middle, and late childhood, subsequently. Throughout these periods, agents are referred to as “children” who do not make economic decisions. They receive time and physical investments to accumulate human capital. Their next three periods are spent as parents and workers. In addition to their human capital, parents are heterogeneous with respect to labor market shocks, asset holdings, and their children’s innate ability. As empty nesters, they spend the next four periods in work and then the remaining two periods in retirement. While the calibration is based on all agents across all periods, I focus on the periods of education investments and human capital formation for policy investigations.

2.1. Formation of Human Capital

Agents entering childhood have heterogeneous innate ability. Parameter $\hat{a}$ measures a child’s productivity of transforming education investments, such as schooling and reading time with parents, into human capital. I assume that through nature, children partly inherit this ability from parents. Given the parent’s ability, $a$, the transmission of innate ability follows an AR(1) process

$$\ln(\hat{a}) = \rho_a \ln(a) + \epsilon_a, \; \epsilon_a \sim N(0, \sigma_a^2),$$

where $\rho_a$ measures the intergenerational persistence of ability and $\epsilon_a$ is an ability shock. With different $\hat{a}$ and private education investments, agents enter adulthood with heterogeneous human capital.

An agent’s human capital, $\hat{h}$, accumulates throughout one’s childhood with a sequential nature. In each of the three periods in childhood, an agent’s human capital depends on current human capital stock and additional education investments. The initial human capital stock is formed by education inputs in early childhood. Accordingly, I specify

$$\hat{h}_1 = \hat{a} \left[ \gamma_1 (h^{3/4})^{\theta_e} + (1 - \gamma_1)e_1^{\theta_e} \right]^{1/\theta_e}.$$

Here, human capital at the end of early childhood $\hat{h}_1$ is a function of innate ability, $\hat{a}$, a time investment component, $h^{3/4}$, and a physical investment component, $e_1$. The time component has two parts, parental human capital, $h$, and parental time investments.
investments, \( t \). Active parenting in early years improves children’s learning ability, so parental time \( t \) contributes to one’s early childhood human capital. Lefebvre and Merrigan (2008) find parental human capital has a positive effect on children’s cognitive development. They argue that this may be from quality parent–children interaction, such as reading and efficient parenting. This argument is supported by Behrman et al. (1999), who suggest that a mother’s education increases the productivity of home teaching. Thus, I allow parents with more human capital \( h \) to be more effective in nurturing their offspring. The \( \delta \) parameter governs this relative effectiveness. The expenditure component \( e_1 \) is a measure of education expenditures made by parents and the government. Here, \( \gamma_1 \leq 1 \) governs the relative weight of time investments, and \( \theta_e \leq 1 \) gauges the substitutability between the two types of investments. It is the specification of \( \hat{h}_1 \) that sets this paper apart from many others. The weight parameter, \( \gamma_1 \), and the substitutability parameter, \( \theta_e \), are the key parameters of interest in estimation.

Early childhood human capital \( \hat{h}_1 \) becomes an input in the production of middle childhood human capital. Parents and the government continue to invest in children and their spending forms a new measure of education expenditures, \( e_2 \). The human capital in middle childhood is created by

\[
\hat{h}_2 = \left[ \gamma_2 \hat{h}_1^{\eta} + (1 - \gamma_2) e_2^{\eta} \right]^{1/\theta_l}.
\]

(1)

Lifetime human capital is determined at the end of late childhood. Its specification is similar to that of middle childhood

\[
\hat{h} = \hat{h}_3 = A \left[ \gamma_3 \hat{h}_2^{\eta} + (1 - \gamma_3) e_3^{\eta} \right]^{1/\theta_l}.
\]

(2)

Here, \( A \) is a scaler that facilitates calibration. Notice that human capital formation in early childhood is distinct in two ways. First, time investment by parents at this stage plays a role. This specification is consistent with the empirical findings that parenting in early years has the greatest impact on children’s development. Second, the elasticity parameter in early childhood \( \theta_e \) captures the interaction between education inputs within this period, while its counterpart in later childhood \( \theta_l \) controls the substitutability among inputs across periods.

The education spending by parents and the government in each period forms a measure of education expenditures equal to

\[
e_i = \left( f_i^\eta + g_i^\eta \right)^{1/\eta}, \quad i \in (1, 2, 3),
\]

(3)

where \( f_i \) and \( g_i \) are private and public spending in the \( i \)th period of childhood, and their substitutability is governed by the parameter \( \eta \leq 1 \). This specification is similar to that in Restuccia and Urrutia (2004) and Arcalean and Schiopu (2010).

### 2.2. Labor Income and Total Output

With accumulated human capital, agents enter the labor market in period four. The wage per unit of time, \( w_j \), depends on one’s human capital, \( h \), experience in the
labor market, \( j \) (\( j \) is discrete and indicates that the worker is in the \( j \)th period of adulthood), and current productivity shock, \( u_j \). More specifically,

\[
w_j = h \exp^{\alpha(j-1)+u_j}, \ j \in (1, 2, \ldots 7).
\]

The experience parameter \( \alpha \geq 0 \) corresponds to the return to experience in the Mincer equation. I allow the productivity shock across periods to follow an AR(1) process

\[
u_{j+1} = \rho u_j + \epsilon_u, \epsilon_u \sim N(0, \sigma_u^2).
\]

Agents are endowed with one unit of time in each period. Time investment in early education affects children’s human capital, so parents choose work hours, \( n \in [0, 1] \) when \( j = 1 \) and spend \( t = 1 - n \) with children. Labor supply is perfectly inelastic and equal to one in the remaining working periods.

Workers in the first period of adulthood differ in both wage and work hours. Their income is \( w_1 n = h \exp^{u_1} n \). Let \( \zeta^{j=1}(h, u_1, n) \) denote the joint probability density function of these variables. Older workers work one unit of time and differ in wage only. Their income is \( w_j = h \exp^{\alpha(j-1)+u_j}, j \in (2, 3, \ldots 7) \). The function \( \zeta^{j>1}(h, u_j, j) \) is defined in a similar way. Total output is linear in wage/human capital. Specifically, it is given by

\[
Y = \int_{H \times U \times N} w_1 n \zeta^{j=1}(h, u_1, n)dhdu_1dn + \sum_{j=2}^{7} \int_{H \times U} w_j \zeta^{j>1}(h, u_j, j)dhdu_j,
\]

where \( H, U, \) and \( N \) represents the range of human capital, productivity shock, and work hours, respectively.

### 2.3. Decision Problem

Adults in the model face different maximization problems in different periods of life. With children in early childhood, adults choose both time and income allocation. As their children enter later childhood, parents work full time and choose consumption, education spending, and savings. The human capital of their offspring is formed by the end of childhood. Parents become empty nesters and only choose consumption and savings thereafter.

Agents receive utility from consumption in each period. They also value the final human capital of their children. The value function of an agent in the \( j \)th period of adulthood is given by \( V_j(.) \). When \( j = 1 \), agents are heterogeneous in human capital, \( h \), the ability of their children, \( \hat{a} \), and current wage, \( w_1 \). Given their current state \((h, \hat{a}, w_1)\), parents choose work hours, \( n \), time with children, \( t \), consumption, \( c_1 \), education spending, \( f_1 \), and bond holdings, \( b_1 \), to solve the following Bellman problem:

\[
V_1(h, \hat{a}, w_1) = \max_{n, t, c_1, f_1, b_1} \left\{ \frac{c_1}{\sigma} + \beta E[V_2(h, \hat{h}_1, w_2, b_1|u_1)] \right\},
\]
subject to
\[ c_1 + f_1 + b_1 = w_1 n (1 - \tau), \]
\[ t + n = 1, \]
\[ e_1 = (f_1^\eta + g_1^\eta)^{1/\eta}, \]
\[ \hat{h}_1 = \hat{a} \left[ \gamma_1 (h^\delta t)^{\theta_e} + (1 - \gamma_1) e_1^{\theta_e} \right]^{1/\theta_e}, \]
\[ u_2 = \rho_u u_1 + \epsilon_u, \epsilon_u \sim N (0, \sigma_u^2), \]
\[ w_2 = h \exp^{\alpha + u_2}, \]
\[ c_1, f_1 \geq 0, \]
\[ 0 \leq n \leq 1. \]

Note that \( \beta < 1 \) is the discount factor and \( \tau \) is a flat income tax. One unit of bond yields \( r \) units of consumption good in the following period. Facing a budget and time constraint, young parents choose \( t \) and \( f_1 \) to form the human capital of their children, \( \hat{h}_1 \). Along with \( h \) and \( b_1 \), \( \hat{h}_1 \) is carried to the next period. The wage in the next period, \( w_2 \), has an uncertain component, \( u_2 \); therefore, agents must have an expectation of future utility.

The value function \( V_2(\cdot) \) is similarly defined. Children’s ability, \( \hat{a} \), is built into their current human capital stock, \( \hat{h}_1 \). So, \( \hat{h}_1 \) replaces \( \hat{a} \) as a state variable. In addition, parents respond to savings from last period, \( b_1 \). The parent’s problem is
\[ V_2(h, \hat{h}_1, w_2, b_1) = \max_{c_2, f_2, b_2} \left\{ \frac{c_2^\sigma}{\sigma} + \beta E[V_3(h, \hat{h}_2, w_3, b_2)|u_2] \right\}, \]
subject to
\[ c_2 + f_2 + b_2 = w_2 (1 - \tau) + b_1 r, \]
\[ e_2 = (f_2^\eta + g_2^\eta)^{1/\eta}, \]
\[ \hat{h}_2 = \left[ \gamma_2 \hat{h}_1^{\theta_i} + (1 - \gamma_2) e_2^{\theta_i} \right]^{1/\theta_i}, \]
\[ u_3 = \rho_u u_2 + \epsilon_u, \epsilon_u \sim N (0, \sigma_u^2), \]
\[ w_3 = h \exp^{2\alpha + u_3}, \]
\[ c_2, f_2 \geq 0. \]

Notice there is no time input in human capital, but parents keep spending on education. Their offspring is in the second period of childhood and a new measure of human capital, \( \hat{h}_2 \), is formed.
Parents carry a similar set of state variables \((h, \hat{h}, w_3, b_2)\) to enter the next period when the Bellman problem is

\[
V_3(h, \hat{h}, w_3, b_2) = \max_{c_3, f_3, b_3} \left\{ \frac{c_3^\sigma}{\sigma} + \frac{\hat{h}^\sigma}{\sigma} + \beta E[V_4(h, w_4, b_3|u_3)] \right\},
\]

subject to

\[
c_3 + f_3 + b_3 = w_3(1 - \tau) + b_2 r, \\
e_3 = (f_3^\eta + g_3^\eta)^{1/\eta}, \\
\hat{h} = \hat{h}_3 = A \left[ \gamma_3 \hat{h}_2^\eta + (1 - \gamma_3) e_3^\eta \right]^{1/\theta_l}, \\
u_4 = \rho u_3 + \epsilon_u \sim N(0, \sigma_u^2), \\
w_4 = h \exp^{3\alpha + u_4}, \\
c_3, f_3 \geq 0.
\]

The scalar \(\xi\) is the altruism parameter that gauges the relative value of children’s human capital. This is the last period for parental education investment. With further private and public spending, \(f_3\) and \(g_3\), children’s final human capital, \(\hat{h}\), is determined.

With children also in the labor market, parents become empty nesters in the fourth period of adulthood. They keep working for another four periods and simply solve

\[
V_j(h, w_j, b_{j-1}) = \max_{c_j, b_j} \left\{ \frac{c_j^\sigma}{\sigma} + \beta E[V_{j+1}(h, w_{j+1}, b_j|u_j)] \right\},
\]

subject to

\[
c_j + b_j = w_j(1 - \tau) + b_{j-1} r, \\
u_{j+1} = \rho u_j + \epsilon_u, \epsilon_u \sim N(0, \sigma_u^2), \\
w_{j+1} = h \exp^{j\alpha + u_{j+1}}, \\
c_j \geq 0, \text{ for } j = 4, 5, 6, 7.
\]

The value functions for retirees are \(V_8(b_7) = \max_{c_8, b_8} \left[ \frac{c_8^\sigma}{\sigma} + \beta V_9(b_8) \right]\) with a budget constraint \(c_8 + b_8 = w_8(1 - \tau) + b_7 r\) and \(V_9(b_8) = \frac{(rb_8)^\eta}{\sigma}. 9\)

2.4. Government

Government taxes labor income with a common tax rate \(\tau\) and allocates the tax revenue toward childhood education. In the baseline model, government funds education through public expenditure in each period of childhood, \(g_1, g_2,\) and \(g_3\).
With a unit mass of children in each education stage, $g_i, i \in (1, 2, 3)$ is both per student and total spending. Thus, the budget relationship is

$$g_1 + g_2 + g_3 = \tau Y,$$

where total output, $Y$, is constructed as in equation (4).

The shares of aggregate output allocated to early, middle, and late childhood education are

$$\varsigma_1 = \frac{g_1}{Y}, \varsigma_2 = \frac{g_2}{Y}, \varsigma_3 = \frac{g_3}{Y}.$$  

The values of $\varsigma_1, \varsigma_2,$ and $\varsigma_3$ are chosen based on the US data. I later investigate the effects of spending level $\varsigma_1$ and consider alternative ways to improve early education, such as education subsidies and paid parental leave. These policies are discussed in detail in the experiment section.

2.5. Equilibrium

Let $S_j$ be the state space for an agent in the $j$th period of adulthood. Notice that $S_j$ may have different dimensions as $j$ varies. A stationary equilibrium is the set of policy functions $n(S_j), t(S_j)$ for $j = 1, f_j(S_j)$ for $j \in (1, 2, 3), c_j(S_j), b_j(S_j)$ for $j \in (1, 2, \ldots 9)$ and the government spending on education, $g_1, g_2,$ and $g_3$ such that

1. agents solve their optimization problems;
2. the government budget is balanced as in equation (5);
3. the bond market is international and accommodates any shortages and surpluses in the goods market;
4. given agents’ optimal decisions and the progression of ability and productivity shocks, the distribution of agents across states is time invariant.

3. CALIBRATION

The model is calibrated to the US economy. For some commonly estimated parameters, I use their empirical counterparts. For the remaining parameters, I choose their values to match a set of moments that characterize some key data features. Table 1 summarizes the values of directly calibrated parameters.

The parameter $\rho_w$ captures the persistence of wage shocks. I choose its value based on Karahan and Ozkan (2013). They find that the persistence of earnings shocks increases with age. Therefore, I choose different values for $\rho_w$ according to an agent’s working stage. Given the length of periods in my model, I adjust the estimates from Karahan and Ozkan (2013) and set $\rho_w = 0.46$ for the first three working periods and $\rho_w = 1$ thereafter.

I set the first preference parameter, $\beta$, to 0.78, which is consistent with the commonly used annual discount rate of 0.96. Empirical estimates for the elasticity parameter $\sigma$ range from $-2$ to 0.5. I choose a middle-range value $\sigma = -0.5$. 


Next, I consider the variables that capture the economic environment. Holter (2015) calculates the average 3-month T-bill rates during period 1947–2008. I take it as the annual interest rate and set \( r = 1.068 \) accordingly. The experience parameter \( \alpha \) captures the wage return of experience. I adopt the estimates by Heckman et al. (2006) and derive \( \alpha = 0.065 \) in the context of this model.

The policy parameters \( \varsigma_1, \varsigma_2, \) and \( \varsigma_3 \) represent the level of government spending in different stages of childhood. Based on the estimates by OECD (2012), I set \( \varsigma_1 = 0.34\% \), \( \varsigma_2 = 1.95\% \), and \( \varsigma_3 = 1.95\% \).

I calibrate the remaining 11 parameters simultaneously. My goal is to match the generated economy to the US economy regarding several general macroeconomic statistics and some key features of parental education investment. Table 2 presents the parameters and their targets. The items in parentheses are the moments targeted by the parameter. The first data column shows the value chosen for the parameter. The last two columns compare the value of the targeted item in the data with that generated by the model.

The first two moments are intergenerational persistence in earnings and consumption. Their values are from Zimmerman (1992) and Mulligan (1997), respectively. Such persistence exists in the model for three reasons. First, innate ability persists across generations. Second, parents with more income spend more in children’s education. Higher education expenditures transform into higher human capital and then higher earnings/consumption. Third, parents can influence their children through quality parenting. The ability persistence parameter \( \rho_a \) governs the first channel and the altruism parameter \( \xi \) strengthens the latter two channels. As in Restuccia and Urrutia (2004) and Holter (2015), I use \( \rho_a \) to match earnings persistence. My choice of \( \rho_a \) falls in the range of their estimates. To target consumption persistence, I use \( \xi \).

The next four parameters largely determine how education inputs from different sources at different stages interact in forming human capital. They are chosen simultaneously to match parental education spending behavior in the United States. The first two moments capture the size of parental spending relative to government spending. Barnett and Masse (2003) use data from the National Household
### TABLE 2. Parameters set endogenously

<table>
<thead>
<tr>
<th>Parameter and target description</th>
<th>Values</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intergenerational persistence of innate ability, $\rho_a$ (Intergenerational persistence of earnings)</td>
<td>0.28</td>
<td>0.44</td>
<td>0.44</td>
</tr>
<tr>
<td>Altruism parameter, $\xi$ (Intergenerational persistence of consumption)</td>
<td>0.43</td>
<td>0.68</td>
<td>0.62</td>
</tr>
<tr>
<td>Weight of time investment, $\gamma_1 \left( \frac{f_1}{g_1+f_1} \right)$</td>
<td>0.25</td>
<td>0.60</td>
<td>0.57</td>
</tr>
<tr>
<td>Weight of h.c. stock in middle childhood, $\gamma_2 \left( \frac{f_2+f_3}{g_2+g_3+f_2+f_3} \right)$</td>
<td>0.84</td>
<td>0.07</td>
<td>0.05</td>
</tr>
<tr>
<td>Substitutability of inputs across periods, $\theta_l$ (Ratio of $f_1$ for second and third income tercile)</td>
<td>-0.45</td>
<td>0.56</td>
<td>0.53</td>
</tr>
<tr>
<td>Substitutability of private and public expenditures, $\eta$ (Ratio of $f_2 + f_3$ for second and third income tercile)</td>
<td>0.35</td>
<td>0.33</td>
<td>0.30</td>
</tr>
<tr>
<td>Substitutability of time and physical inputs, $\theta_e$ (Wage elasticity of time investment)</td>
<td>-0.90</td>
<td>0.13</td>
<td>0.14</td>
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<td>Curvature parameter for time investment, $\delta$ (Average time investment)</td>
<td>0.36</td>
<td>0.17</td>
<td>0.18</td>
</tr>
<tr>
<td>s.d. of wage shocks, $\sigma_w$ (Gini index)</td>
<td>0.43</td>
<td>0.44</td>
<td>0.43</td>
</tr>
<tr>
<td>s.d. of ability shocks, $\sigma_a$ $\left( \frac{\text{var}(\ln h)}{\text{var}(\ln \text{income})} \right)$</td>
<td>0.53</td>
<td>0.60</td>
<td>0.58</td>
</tr>
<tr>
<td>Scalar of h.c., $A$</td>
<td>6.9</td>
<td>n.a.</td>
<td>n.a.</td>
</tr>
<tr>
<td>Government budget balance</td>
<td></td>
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</tr>
</tbody>
</table>

Education Survey and show that about 60% of early childhood spending is private. This ratio is larger than the estimate of 19% from OECD (2012). This is because Barnett and Masse (2003) include household spending on child care, while OECD (2012) only considers formal education. Given the context of my model, targeting $\frac{f_1}{g_1+f_1} = 0.6$ is more appropriate. Regarding its counterpart in later childhood, I use data from OECD (2012) and target $\frac{f_2+f_3}{g_2+g_3+f_2+f_3} = 0.07$. The other two moments are from U.S. Department of Agriculture (2013). This study reports the education spending by parents in different income groups. I use the data for families in the middle and high terciles only, considering that the government may subsidize low-income families to defray child care expenses.

The level of education spending in each period partly depends on its relative importance. Therefore, I use $\gamma_1$ to target parental spending in early childhood relative to total spending, $\frac{f_1}{g_1+f_1}$. Similarly, parameters $\gamma_2$ and $\gamma_3$ are related to $f_2$ and $f_3$. To see how investments in middle and late childhood contribute to the final human capital, I plug equation (1) into equation (2):

$$\hat{h} = A \left[ \gamma_2 \hat{h}_1^{\eta_2} + \gamma_3 (1 - \gamma_2) e_2^{\eta_2} + (1 - \gamma_3) e_3^{\eta_3} \right]^{1/\theta_l}.$$ 

Here, the coefficients sum to one and they represent the weight of early childhood human capital, education expenditures in middle and late childhood, respectively. This model explores the distinct role of early education, so I assume that
expenditures in the latter two periods are equally important, while $\hat{h}_1$ can be more productive. This leads to $\gamma_3(1 - \gamma_2) = (1 - \gamma_3)$. With this assumption, I pin down $\gamma_2$ with relative parental spending in later childhood $\frac{f_2 + f_3}{g_2 + g_3 + f_2 + f_3}$ and recover $\gamma_3$. My estimates of the $\gamma$ values show that investments in the early stage are more productive.\(^{12}\)

The substitutability parameters $\theta_l$ and $\eta$ are used to target the distribution of parental spending. In general, parents with more income spend more on children’s education. As $\theta_l$ increases, education inputs are more substitutable across periods, and thus government spending in later childhood can compensate better for lower spending in early childhood by poor parents. This serves to widen the gap in parental spending on young children. The substitutability between parental and government spending, $\eta$, has an analogous impact on parental spending ratios. My estimate of $\theta_l$ is negative, indicating a relative complementarity among investments across periods.\(^{13}\) To explain this common finding, Heckman (2006) argues that acquired skills and abilities in early years motivate children to learn more and make later learning more efficient. My choice of $\eta$ implies that private and public spending are somewhat substitutable. This setting is common in the literature.\(^{14}\)

I proceed to choose two key parameters for the role of time investment, $\theta_e$ and $\delta$. The substitutability parameter $\theta_e$ gauges the relationship between time investment and wage. A higher value of $\theta_e$ indicates that time input can be more easily substituted by education spending, and therefore, it results in less time spent by rich parents. So, I choose $\theta_e$ to target the wage elasticity of time input. This moment is obtained using data from the American Time Use Survey (2013).\(^{15}\) The ATUS collects data on the amount of time people spend doing various activities, including work, child care, recreation, etc. There are three categories for time spent with/for children: (1) caring for and helping children; (2) doing activities related to children’s education; (3) doing activities related to children’s health. I measure time investment by summing the variables in all three categories. The wage rate is calculated by dividing weekly earnings by weekly work hours. The target of elasticity is well matched when $\theta_e = -0.9$, suggesting a complementarity between time and goods investments.\(^{16}\)

Parameter $\delta$ regulates the productivity of parental human capital. As $\delta$ increases, all parents become more efficient and thus may increase parental time. However, with a higher level of time component in the production, its marginal return decreases and that may result in less parenting. With the chosen values of other parameters, the latter effect dominates in the benchmark, and therefore, a higher $\delta$ corresponds to a lower $t$. Due to this relationship, I use $\delta$ to target average time investments. Based on data from ATUS (2013), parents with young children spend, on average, 2.08 hours per day on child care.\(^{17}\) I assume that total hours available for work and children is 12 per day and target $\bar{t} = 0.17$.\(^{18}\)

The next two parameters are related to the shocks in the economy. I choose the s.d. of wage shocks to target the Gini index of 0.44.\(^{19}\) For the s.d. of ability shocks, I match the ratio of variation in human capital and lifetime income,
TABLE 3. Untargeted moments

<table>
<thead>
<tr>
<th>Untargeted moments</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{t} ) in the low-wage tercile</td>
<td>0.157</td>
<td>0.160</td>
</tr>
<tr>
<td>( \bar{t} ) in the middle-wage tercile</td>
<td>0.172</td>
<td>0.175</td>
</tr>
<tr>
<td>( \bar{t} ) in the high-wage tercile</td>
<td>0.188</td>
<td>0.189</td>
</tr>
<tr>
<td>Ratio of ( f_1 ) for first and third income tercile</td>
<td>0.393</td>
<td>0.289</td>
</tr>
<tr>
<td>Ratio of ( f_2 + f_3 ) for first and third income tercile</td>
<td>0.202</td>
<td>0.100</td>
</tr>
<tr>
<td>Variance (ln income)</td>
<td>0.360</td>
<td>0.503</td>
</tr>
</tbody>
</table>

\[ \frac{\text{var}(\ln h)}{\text{var}(\ln \text{income})} = 0.6. \] This moment is from Huggett et al. (2011). They find that about 60% of the variation in earnings is due to the variation in predetermined worker characteristics, such as education.

In steady state, government spending is at the level that generates the same aggregate human capital/output as the previous period. Given the construction of total output in equation (4), the budget constraint in equation (5) is also

\[ g_1 + g_2 + g_3 \int_{H \times U \times N} w_1 n \xi j=1(h, u_1, n)dhdu_1dn + \sum_{j=2}^{7} \int_{H \times U} w_j \xi j=1(h, u_j, j)dhdu_j \]

The tax rate \( \tau = \varsigma_1 + \varsigma_2 + \varsigma_3 = 4.24\% \), which pins down the size of government spending relative to final human capital in steady state. While this ratio is affected by many parameters, it is directly linked to productivity parameter \( A \).

Overall, the model does a good job reproducing the targeted moments. For further validation of the model identification, I perform an exercise to check how the model fits data with some untargeted moments. Table 3 presents the results. Notice the model maintains monotonicity for time investment in wage, and the average time spent with children in each wage group is relatively well matched. The education spending ratio at each stage is slightly smaller in the model than in the data. This is expected, because the government subsidizes low-income families in real life to boost their spending on children while there is no government subsidy in the benchmark economy. The moment for income inequality is from Mulligan (1997). He reports the variance of the log of earnings equal to 0.36. This moment is 0.503 in the model. In general, the model is successful in replicating some untargeted features of education investment as well as earnings variation.

4. POLICY EXPERIMENTS

In this section, I consider three policy experiments regarding early education and compute the stationary recursive equilibrium in each of the counterfactual economies. The first exercise is to assume the government doubles public spending in early childhood. The second policy is to subsidize parental spending and the third is to provide partially paid parental leave. All the policies are financed by a
TABLE 4. Experiment results

<table>
<thead>
<tr>
<th></th>
<th>Benchmark</th>
<th>Increasing (g_1)</th>
<th>Subsidizing (f_1)</th>
<th>Paid parental leave</th>
</tr>
</thead>
<tbody>
<tr>
<td>Human capital</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Investment in early education</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Private expenditures, (f_1)</td>
<td>0.554</td>
<td>0.473</td>
<td>0.582</td>
<td>0.649</td>
</tr>
<tr>
<td>Public expenditures, (g_1)</td>
<td>0.425</td>
<td>0.850</td>
<td>0.425</td>
<td>0.425</td>
</tr>
<tr>
<td>Time investment, (t)</td>
<td>0.175</td>
<td>0.183</td>
<td>0.184</td>
<td>0.197</td>
</tr>
<tr>
<td>Other aggregate statistics</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Earnings persistence</td>
<td>0.441</td>
<td>0.403</td>
<td>0.422</td>
<td>0.431</td>
</tr>
<tr>
<td>Var. of log earnings</td>
<td>0.503</td>
<td>0.475</td>
<td>0.484</td>
<td>0.493</td>
</tr>
</tbody>
</table>

flat income tax. For comparison purposes, I choose the subsidy rates in the latter two experiments endogenously so that the cost of all policies is constant.

4.1. Increasing Government Spending

I first consider doubling government expenditures in early childhood while keeping government expenditures in later childhood the same. In this case, \(g_1 = 0.0068Y\), \(g_2 = 0.0195Y\), and \(g_3 = 0.0195Y\), where \(Y\) is aggregate output in the benchmark economy. To fund the extra spending, tax rate \(\tau\) is raised from 0.0424 to 0.0458.20

The first data column of Table 4 shows the benchmark case and average human capital is normalized to 1. The next three data columns present the experiment results. Public spending in early childhood has a crowding out effect on private spending. When government spending doubles, private spending is reduced from 0.554 to 0.473. However, total education expenditure increases and it results in a higher level of time investment due to their complementarity. As a result, human capital stock in early childhood increases by 16%, and this increase is transformed into a 12.5% increase in final human capital.

Earnings persistence drops to 0.403 from 0.441. In general, parents with more income spend more on education. Such parental income/education spending relationship is a source of earnings persistence. In the benchmark economy, poor parents spend almost nothing on children’s early education, so there is little room for government spending to crowd out parental spending. In contrast, rich parents spend substantially more on education, and more crowding out occurs. For example, families in the lowest income tercile decrease early education spending by 0.04, while this number is 0.13 for families in the highest income tercile. The result is a weaker link between parental income and education spending, leading to less persistence.

Income inequality, as measured by the variance of log earnings, decreases from 0.503 to 0.475. This change is caused by different crowding out effects across families, as discussed above. The same increment of government spending results
4.2. Subsidizing Parental Spending

The same public resources can be allocated to early education by subsidizing parental spending. With a fixed subsidy rate \( s_f \in [0, 1] \), the government transfers \( s_f f_1 \) to parents for education uses. So, equation (3) for \( i = 1 \) can be rewritten as

\[
e_1 = \left( (f_1 + s_f f_1)^\eta + g_1^\eta \right)^{1/\eta}.
\]

The state space for agents in early parenthood is \( S_1 = (h, \hat{a}, w_1) \). Let \( \kappa_1(h, \hat{a}, w_1) \) denote the joint probability density function of these state variables. The total subsidy is

\[
TS = s_f \int_{H \times \hat{A} \times W} f_1 \kappa_1(h, \hat{a}, w_1) dhd\hat{a}dw_1. \tag{6}
\]

Parental spending responds to the subsidy rate endogenously, so the cost of this policy responds to a unique level of \( s_f \). In the previous experiment, the increment in government expenditures is 0.0034Y. To generate comparable policy effects, I choose \( s_f = 0.726 \) in the current experiment to target \( TS = 0.0034Y \).

The subsidy increases private spending. A dollar increase in private spending now induces more than a dollar increase in education investment. With this higher return, all parents spend more. This effect is smaller among the rich, given their higher spending level and lower marginal return. Under the subsidy, parents in the lowest earnings tercile increase spending by twice as much as parents in the highest earnings tercile. On average, private spending in early childhood increases from 0.554 to 0.582. With more private spending and the same increment in government spending, the subsidy generates a larger increase in human capital than direct government spending. The average human capital in the new equilibrium increases by 14.4%.

While the subsidy has a larger impact on aggregate human capital than direct government spending, two policies have different distributional effects. Both earnings persistence and inequality decrease again, but the changes are smaller. A subsidy transfer of a fixed value has a higher return to families with lower spending, which helps to narrow the income gap within and across generations. However, rich families receive a larger subsidy transfer under current policy, weakening the effects on earnings inequality and persistence.

4.3. Paid Parental Leave

In this section, I explore the channel of compensating parents for investing time in children. The compensation is proportional to parents’ lost earnings. Let \( s_l \in [0, 1] \) be the percentage of forgone earnings replaced by the government. The budget
constraint for young parents becomes
\[ c_1 + f_1 + b_1 = w_1 n(1 - \tau) + s_l w_1 t. \]

Similar to equation (6), the total compensation under this program is
\[ TC = s_l \int_{H \times A \times W} w_1 t z_1(h, \hat{a}, w_1) \, dh \, d\hat{a} \, dw_1. \]

Any given \( s_l \) is strictly mapped to a TC. Again, I choose the value of \( s_l \) to target \( TC = 0.0034Y \), so the government allocates the same amount of extra resources to this program as it does to the alternatives. I find that when parents on leave receive 13.4% of their forgone earnings, total public expenditure in early childhood doubles. Therefore, the policy parameter setting in this experiment is \( s_l = 0.133 \).

Compared to previous policies, paid parental leave directly targets time investment rather than physical investment. The current program leads to the largest increase in time input. More specifically, parents, on average, spend 12.6% more time with their young children. This relationship between the presence of a paid leave program and less work hours is documented by many empirical studies.\(^{21}\) The increment in time input triggers an increment in private spending, and together they generate an increase in aggregate human capital of 11.3%.

While it creates the largest increase in parental time, the leave program is least effective in promoting human capital. This is because the extra public resources are not directly assigned for education use as in the first two programs. Parents receiving the compensation allocate the subsidy between consumption and education spending. The results show only 23.5% of the total subsidy is spent on education.

Rich parents take more time off for child care and have higher wages, therefore their forgone earnings is higher and so is their compensation. So the leave program allocates more additional resources to the rich. Therefore, its impact on earnings persistence and inequality is relatively small.

4.4. Discussion

In this section, I compare the welfare effects of policies discussed above. The welfare gain for each agent is defined as the percentage change in consumption in the benchmark economy that generates the same utility as when a policy is implemented. The lifetime utility of an agent in the baseline economy is
\[ U_b = \sum_{j=1}^{7} \beta^{j-1} \left( \frac{c^b_j}{\sigma} \right)^{\sigma} + \beta^2 \xi \left( \hat{h}^b \right)^{\sigma}, \]

where \( c^b_j \) is the consumption level in one’s \( j \)th working period in the baseline economy and \( \hat{h}^b \) is the final human capital of one’s offspring. For any alternative
economy (with more direct government spending, an education subsidy, or a paid parental leave), the utility of the same agent can be written as

\[ U_a = \sum_{j=1}^{7} \beta^{j-1} \left( \frac{c_j^a}{\sigma} \right)^\sigma + \beta^2 \xi \left( \frac{h^a}{\sigma} \right)^\sigma. \]

An agent will be indifferent between the baseline economy and an alternative economy if his utility levels are equal. So the welfare gain is the \( \Delta c \) that solves the following equation:

\[ \sum_{j=1}^{7} \beta^{j-1} \left[ \frac{c_j^b (1 + \Delta c)}{\sigma} \right]^\sigma + \beta^2 \xi \left( \frac{h^b}{\sigma} \right)^\sigma = \sum_{j=1}^{7} \beta^{j-1} \left( \frac{c_j^a}{\sigma} \right)^\sigma + \beta^2 \xi \left( \frac{h^a}{\sigma} \right)^\sigma. \]

The average welfare gain for the whole economy is the percentage change in consumption for all agents in the baseline economy that gives the same aggregate utility as the counterfactual economy. To better compare the distributional policy effects, I calculate the average welfare gain for agents in each tercile of lifetime earnings. Figure 1 shows the results.

In general, poor parents benefit most from all three policies and this trend is most prominent under direct spending. For parents in the lowest, middle, and highest income tercile to be indifferent between the benchmark economy and an economy with doubled government spending on early education, they need an increase in consumption of 19.5%, 15.7%, and 11.5%, respectively. An education subsidy with the same cost is equivalent to an increase in consumption of 20.1%, 17%, and 14.5% to them. The welfare gain under parental leave is the smallest for all groups. They are 15%, 13.2%, and 11%.

The policy that doubles government investment benefits the poor substantially more due to the large crowding out effect among the rich. An education subsidy allocates more resources to parents with more education spending and thus the gap in welfare gain is smaller. As illustrated in Figure 1, the education subsidy has the largest welfare impact on all, but its effect on inequality is smaller than direct spending. The parental leave policy is the most expensive means of improving welfare. The policy increases welfare by partially correcting the distortion in human capital accumulation. While the leave program provides parents incentives to spend time with their children, a large part of the wage replacement is used for consumption. As a result, the welfare gain is relatively small from this program.

To this point, I have established that all three policies generate welfare gains that disproportionately help the poor. This result motivates another set of experiments. I implement the same policies but this time extra resources only go to the lowest income tercile. I compare these means-tested policies with their broad-based counterparts in Table 5. The first data column reiterates the main statistics in the benchmark. The next three columns show the outcomes of means-tested policies using numbers with a star. I also include the results from earlier experiments in parentheses for comparison purposes. Notice that current policies all result in a
FIGURE 1. The three bars in the first section show the average welfare gain for agents in the lowest tercile of permanent income with doubled government spending, an education subsidy, and a paid parental leave, respectively. The second and third sections show their counterparts for agents in the middle and highest income tercile.

TABLE 5. Experiment results of means-tested policy

<table>
<thead>
<tr>
<th></th>
<th>Benchmark</th>
<th>Increasing $g_1^*$</th>
<th>Subsidizing $f_1^*$</th>
<th>Paid parental leave*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Human capital</td>
<td>1</td>
<td>1.183*(1.125)</td>
<td>1.099*(1.144)</td>
<td>1.111*(1.113)</td>
</tr>
<tr>
<td>Earnings persistence</td>
<td>0.441</td>
<td>0.367*(0.403)</td>
<td>0.378*(0.422)</td>
<td>0.379*(0.431)</td>
</tr>
<tr>
<td>Var. of log earnings</td>
<td>0.503</td>
<td>0.475*(0.475)</td>
<td>0.462*(0.484)</td>
<td>0.462*(0.493)</td>
</tr>
</tbody>
</table>

lower earnings persistence and less income inequality. This is expected as they are designed to help the poor.

When targeting the poor, government spending has a larger impact on aggregate human capital. This result confirms the earlier insight into crowding out. Since there is little room for crowding out in poor families, they are better at converting extra spending into human capital. As a result, aggregate human capital increases
by 18.3%, rather than 12.5%. In contrast, a spending subsidy generates more human capital when it is applied to all. With the complementarities among education inputs, one needs to increase time input as well as spending in later childhood to better take advantage of the subsidy. Well-endowed parents are better able to respond this way. The paid leave policy generates a similar effect as before. When the paid leave is only available to the poor, the wage replacement rate is more than doubled. The eligible group is relatively responsive to this change and their human capital gain is much larger than before. This allows the aggregate effect to remain nearly constant.

4.5. Sensitivity Check

The calibration results show strong intraperiod (within early childhood) and interperiod (between early and later childhood) complementarities among education inputs. This is captured by the parameters $\theta_e = -0.9$ and $\theta_l = -0.45$, respectively. To investigate how these parameters affect the policy predictions, I perform a sensitivity check. The first column of Table 6 shows the alternative values I consider. I normalize the average human capital in each of the five cases to 1 and report their counterparts under each of the three policies in columns 2–4.

<table>
<thead>
<tr>
<th>Parameter Configuration</th>
<th>Increasing $g_1$</th>
<th>Subsidizing $f_1$</th>
<th>Paid parental leave</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline ($\theta_e = -0.9, \theta_l = -0.45$)</td>
<td>1.125</td>
<td>1.144</td>
<td>1.113</td>
</tr>
<tr>
<td>$\theta_e = -0.5$</td>
<td>1.215</td>
<td>1.182</td>
<td>1.163</td>
</tr>
<tr>
<td>$\theta_e = 0$</td>
<td>1.170</td>
<td>1.039</td>
<td>1.036</td>
</tr>
<tr>
<td>$\theta_e = 0.5$</td>
<td>1.074</td>
<td>1.039</td>
<td>1.005</td>
</tr>
<tr>
<td>$\theta_l = 0$</td>
<td>1.116</td>
<td>1.153</td>
<td>1.139</td>
</tr>
<tr>
<td>$\theta_l = 0.5$</td>
<td>1.082</td>
<td>1.136</td>
<td>1.100</td>
</tr>
</tbody>
</table>

In the first three cases, I consider a higher value for $\theta_e$, indicating more substitutability between goods and time input. The first four rows show a common trend with all policies. The impact on aggregate human capital first increases and then decreases with $\theta_e$. As $\theta_e$ starts to increase, time and goods input become more substitutable in producing human capital in early childhood. There is less of a need for parents to match targeted input for a policy to be effective. Therefore, a program that targets either input becomes more productive. However, as $\theta_e$ continues to increase, the two education inputs get closer to becoming perfect substitutes and crowding out occurs. For example, when $\theta_e = -0.9$, doubling government spending, $g_1$, causes an increase in parental time, $t$, by 4.6%. In contrast, when $\theta_e = 0.5$, parents respond to such an increase in spending with a reduction of time input by 22.9%. The effect of crowding out eventually dominates and reduces the overall effectiveness of policies.
Notice the policy effect of the parental leave program decreases most drastically. The change in human capital is reduced to 0.5% with $\theta_e = 0.5$. At a such high level of $\theta_e$, parents spend most of their time working and education expenditure becomes the main input for human capital. The program that directly targets time input appears to be ineffective in this scenario.

In the following two cases, I allow early and late investment to be more substitutable with $\theta_l = 0$ and $\theta_l = 0.5$. Again, policy effects eventually decrease with $\theta_l$ for an analogous reason. However, even with $\theta_l = 0.5$, all policies remain effective with an increment in human capital of 8.2%, 13.6%, and 10%, respectively. The key to understanding this is the spending pattern in later childhood. Parental spending accounts for only 7% of total spending at this stage and $f_2 = f_3 = 0$ for many families. This leaves little room for crowding out when the investment level increases in early childhood. Therefore, early education policies stay effective even if interperiod substitutability is high.

5. CONCLUSION

Parent–child interactions are considered to be vital in early childhood. Quality interactions with parents improve children’s focus, foster trust, and build a foundation for intellectual curiosity and active learning. This paper contributes to the literature by investigating the role of parental time investment in early childhood.

I build a life-cycle overlapping generations model where human capital is formed in early and later childhood. The model is calibrated to the US economy so that the generated data matches parental involvement in childhood education and several aggregate statistics. Starting from the benchmark economy, I consider more public spending, an education subsidy, and paid parental leave.

I find that parental time is complementary to goods investments. However, a paid family leave program is least effective in increasing human capital. While the program has a large and positive impact on parental time investments, much of the wage compensation is used on consumption, rather than education. In contrast, doubling government spending and subsidizing private spending each allocate all the additional resources to early education. In addition, parents voluntarily spend more time with their kids under these policies as the return to their time increases. This finding is of interest to policy makers as many advocates of paid parental leave are calling for new state and federal policies. While paid parental leave may be desirable for other reasons, my study indicates that it is not the most efficient means of achieving higher human capital. Between the two relatively efficient policies, the subsidy program generates a larger increment in aggregate human capital than increased government spending. However, this larger impact on output is accompanied by a smaller reduction in earnings persistence and inequality, demonstrating the efficiency–equity trade-off.
This paper considers time allocation choice as the only determinant of employment. However, demand forces in the labor market may also be affected by the adoption of paid parental leave. An extension of this research might be a more comprehensive analysis that includes the responses of employers. I also abstract from fertility choices by assuming one child is born into each family. Incorporating endogenous fertility choices would create another interesting layer of policy analysis. I leave these extensions for future research.

NOTES

2. See Cunha et al. (2005), Cunha and Heckman (2007), and Caucutt and Lochner (2011).
3. Such expenditures are used for day care and physical tools to help children learn, such as toys, games, books, and software.
4. See Kalenkoski et al. (2005) and Lefebvre and Merrigan (2008).
5. See Baker et al. (2008) and Gupta and Simonsen (2010).
6. Morchio (2013), Del Boca et al. (2014), Brilli (2017) estimate a Cobb–Douglas production function, assuming the substitutability parameter is 0; Griffen (2012) has a value-added production function implying perfect substitution between inputs; Yum (2016) excludes goods input from the production function.
7. Other than the shocks to innate ability \( a \), there are no idiosyncratic shocks in the formation of human capital. This setting allows me to focus on the question of interest and to keep the computational cost under control.
8. Notice that experience is only related to the period of life a worker is in. It is not affected by his time invested in children.
9. I abstract away the retirement benefit. It is less significant for the question at hand, as fertility and population are not endogenous in the model.
10. See Keane and Wolpin (2001) and Beaudry and Wincoop (1996).
11. In Restuccia and Urrutia (2004), \( \rho_a = 0.2 \) and in Holter (2015), \( \rho_a = 0.332 \).
12. This feature is consistent with the findings of Cunha et al. (2005) and Knudsen et al. (2006).
13. This is supported by Caucutt and Lochner (2011) and Cunha et al. (2010).
15. Wage elasticity of time investment is estimated by regressing the logarithm of \( t \) on the logarithm of the wage rate. I only include observations with the following characteristics: (1) respondents older than 21 and younger than 60; (2) married; (3) the youngest child in the household younger than 6; (4) weekly work hours greater than 0; (5) earnings information available; (6) time spent with children greater than 0.
16. Zhu and Vural (2013) calibrate a parameter of this sort and choose \(-0.91\) for its value.
17. The same criterion is applied in choosing observations for the mean of \( t \) as for the wage elasticity of \( t \).
18. In the estimation of the wage elasticity of time and the average time input, I do not account for the endogenous response of the spouse. Guryan et al. (2008) control for marital status and show that child care time ranges between 1.73 and 2.43 hours per day. Kimmel and Connelly (2007) control for marital status and spousal earnings and report a wage elasticity of 0.48. In addition to the three categories I count as child care time, they also include time spent engaging nonparental caregivers and transportation time to nonparental care arrangements. My estimate of 0.13 is closer to the number adopted by Zhu and Vural (2013). Their calculation is similar to mine and use 0.207 as their target.
19. The Gini coefficient is from the Organization for Economic Co-operation and Development (OECD), an estimate before taxes and transfers, for the working age population (18–65) around the year 2000.
20. Notice that in the new steady state, total output is higher so as total tax revenue. With the same education spending, the government runs a budget surplus under all new policies.

21. See Baker and Milligan (2008), Burgess et al. (2008), and Dustmann and Schonberg (2008).

22. To facilitate the comparison, the cost of means-tested policies is kept the same as the broad-based programs. Therefore, the spending level, the subsidy rate, and the wage replacement rate are all adjusted accordingly.

23. The values of remaining parameters and the level of government spending are held the same. The economy reaches a new steady state with a different level of GDP. Therefore, the government budget is not balanced with alternative values of $\theta_e/\theta_l$.

24. This issue was brought to light when several large companies announced to offer paid parental leave. Adobe, Microsoft, Facebook now offer paid leave between 16 and 26 weeks.

REFERENCES


