Early childhood education expenditures and the intergenerational persistence of income*

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Abstract

We consider the extent to which cross-country differences in the intergenerational persistence of income can be explained by differences in government spending on early childhood education. We build a life-cycle model where human capital is accumulated in early, middle and late childhood. Both families and the government can increase the human capital of young agents by investing in education at each stage of childhood. Ability in each dynasty and wages per unit of human capital are stochastic. Different realizations of these values and the resultant education spending histories generate a stochastic steady-state distribution of income. Government spending can reduce persistence by weakening the link between parental income and education spending for a child. Our results show that doubling early childhood spending in the U.S. to match levels in Norway and Denmark eliminates less than 8.5 percent of the gap in intergenerational income persistence. Increased government education spending in later childhood has almost no effect on persistence. Early childhood expenditures can have a larger effect when allocated to low income families.

JEL Classification: E62, I22, H52, J24

Keywords: Government education expenditures, life-cycle model, early childhood education, intergenerational persistence of earnings

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1 Introduction

Children from lower income families tend to earn less in adulthood than children from wealthier families. This reflects more than inherited traits. Children from wealthier families are provided with more and better education, a socioeconomic environment more suitable for human capital accumulation, and greater workplace opportunities through networking. As children transition to the labor market, these accumulated differences result in an intergenerational persistence of income.

Compelling evidence demonstrates that skills attained early in life form the foundation of later achievement. Cunha et al. (2005) show that disparities in ability across young children account for much of the variation in socioeconomic outcomes as adults. Knudsen et al. (2006) cite evidence from economics, neurobiology and sociology to show that different abilities and skills are formed in different stages of the life cycle and that some essential skills are developed in early childhood.\(^1\) Since early childhood education plays a foundational role in human capital development, families can transfer earning potential to their children through this channel.

Public funding of early childhood education can lead to more uniformity in early learning opportunities. Countries differ widely in the extent of such funding. Government spending on early childhood education is equal to 0.4 percent of GDP in the United States. Norway and Denmark spend twice this amount. Lower government spending in the U.S. means that resources available in the critical years of early childhood are more closely related to parental income.

Income persistence is much higher in the U.S. than in Denmark and Norway. While persistence measures are only 0.17 in Norway and 0.15 in Denmark, it is equal to 0.47 in the U.S. The differences in funding and persistence across these countries suggest an opportunity for the U.S. to lower the persistence gap through lowering the funding gap. This paper considers the extent to which the persistence gap could be narrowed if the U.S. were to adopt the funding level for early childhood education seen in Norway and Denmark. Despite the central role of early childhood education, we find little scope for reducing persistence through government education spending. Doubling government funding of early childhood education in the U.S. to match spending in these Nordic countries would eliminate less than 8.5 percent of the observed gap in persistence. Moreover, we find that increased funding for children in later childhood has almost no effect on persistence. The explanation for large differences in persistence apparently lies beyond differences in government funding of education in childhood.

\(^1\) Related work includes Carneiro and Heckman (2003) and Currie (2001).
We investigate the relationship between government early education spending and the inter-generational persistence of income using a stochastic, heterogenous agent life-cycle model. We follow Cunha et al. (2010) and model human capital accumulation as a multi-stage process where the timing of education investment is critical to its effectiveness. The education outcome at each stage depends on the education outcome at the previous stage, parental human capital, and current investment. We model early childhood education as being both relatively productive and a complement to later childhood education.

In our model, government education spending serves to weaken the link between total education spending and family income. Families respond to government spending in part by reducing private spending. For poor families, private spending is low and there is scarce room for such crowding out. An increment to government spending then lowers private spending more for the wealthy than for the poor. Equivalently, increments to government expenditures increase total spending more for low income families than for high income families. As a result, income and education spending are less closely linked and the persistence of income diminishes. Our results show that the opportunity to reduce persistence through spending on early childhood education is largely exploited at current funding levels. The current level of spending has a relatively large effect on persistence. Eliminating government spending on early education would increase persistence by nearly 18 percent. In contrast, an equivalent increase in spending would decrease persistence by just over 5 percent.

In part, this relatively small impact reflects a weakening of the crowding out effect. At higher levels of government spending, families provide a smaller share of total spending. This leaves less room for additional crowding out and additional equalization of spending. At higher levels of spending, then, a second effect on persistence has a relatively larger impact. Since the crowding out is incomplete, increased government spending results in increased total spending for all agents. The same amount of increase in spending has a larger effect on the offspring of the wealthy since on average these are the more able students. This second effect serves to strengthen the relationship between parent and progeny income.

The decreasing marginal impact of government education spending on persistence is even more clear at the primary and secondary levels. Current spending levels on primary and secondary education in the U.S. are much larger than on early childhood education. We show that further spending at these levels has almost no effect on persistence.
An increase in early childhood education spending can be more effective when targeted at the lower end of the income distribution. We show that an equivalent increase in government spending would decrease persistence by 17 percent, rather than 5 percent, if it were allocated only to the lowest income quintile. This is because total spending for affected families increases nearly in proportion to government spending while total spending for wealthier families is unaffected. As a result the gap in spending falls. This is helpful in understanding the impact of such programs as the Perry Preschool Project, the Abecedarian Project (see Cunha and Heckman (2007)), and Head Start (see Currie (2001)). These are programs targeted directly at the early development of children from low income families and each has arguably been highly effective.

Restuccia and Urrutia (2004) also consider the role of education at different stages on the intergenerational persistence of earnings. However, they focus on a two-stage education process where early education encompasses all of education prior to college. Features that distinguish these two levels of education are quite different than those that distinguish early and later childhood education in our paper. Thus they consider related but distinct questions. They find that increasing funding of pre-college education is more effective than funding for college. In this sense our work can be seen as a refinement of this prescription. We argue that when increasing pre-college funding, it is best to focus these additional resources on the pre-primary period.

Holter (2014) builds a model in a similar vein in order to understand better the sources of differing levels of intergenerational income persistence in Western economies. He also considers how persistence in the U.S. would change upon implementing Danish taxation and government education spending policies. He finds that the required increased progressivity of taxes would have a larger effect on persistence than the required spending changes. Holter models education as a multi-stage process. While there are several periods prior to college, education is assumed to begin at age 5 and the pre-college periods contribute to human capital in a symmetric fashion. Also, there are not sharp funding differences across the pre-college periods. Thus the paper addresses a distinct but complementary set of questions.

Our work also relates to Cunha (2013). Cunha models human capital accumulation in childhood as a multi-stage process and adopts similar inputs in forming human capital. As in our model, he allows different returns to education inputs in early and later childhood. However, there are two distinctions between our model and his. First, innate ability is omitted in his model, while we include it as an input in human capital production and specifically model its intergenerational
transference. Second, Cunha estimates the role of parental investment based on measurements of home environment and planned learning activities. In contrast, we treat parental investment as education spending and calibrate the related parameters to replicate family spending behaviors in the U.S. These differences allow us to focus on the relationship between intergenerational persistence and government funding.

Our work is also related to Abington and Blankenau (2013) and the model in that paper is the starting point for this work. Abington and Blankenau consider circumstances under which the current government funding structure, i.e. focussing on later childhood, can be appropriate despite the importance of early childhood education. The paper doesn’t consider income persistence. Agents perfectly inherit the ability of their parents. In a steady state, children’s income is the same as parental income so persistence is equal to one. Substantial modifications to the model are required to facilitate an investigation of income persistence. These include the stochastic process for wages and skill inheritance and a more developed specification for human capital accumulation. Moreover, calibrating the model for our experiments requires a more complex life-cycle structure.

The paper proceeds as follows. In Section 2, we present the model. In Section 3 we describe the calibration strategy and report how well the model fits U.S. data. In Section 4, we discuss the effects of current policy and the driving forces in the model. We then run the policy experiments and discuss our main results in Section 5. In Section 6 we provide a summary and conclusion.

2 The model

We consider an overlapping generations model where agents live for twelve periods and each period lasts 6 years. The first period is spent in early childhood, the next two in middle and late childhood, the fourth as a worker and the parent of a child in early childhood, the next two as a worker and the parent of a child in middle and then late childhood, and the remainder in work and then retirement as empty nesters. Each generation consists of a continuum of agents, each belonging to a distinct dynasty. The mass of each generation is normalized to one. Each dynasty has a child every three periods. The dynasties are staggered so that a child is born into one third of the dynasties in each period. At the beginning of any period $t$, a continuum of agents comprising generation $t$ are born into early childhood as the prodigy of the current young parents. In the subsequent two periods, as their parents move to middle and late parenthood, the offspring move to middle and late childhood. Generation $t$ reaches parenthood in period $t+3$ and have offspring who are in early childhood. As
they transition to middle and late parenthood in periods $t + 4$ and $t + 5$, their children transition to middle and late childhood. As period $t + 6$ begins, they are empty nesters and continue working through period $t + 9$. In periods $t + 10$ and $t + 11$ they are retired and subsequently exit the economy. The twelve period structure is convenient for our calibration. However, the life stages of human capital accumulation and education spending (periods $t$ through $t + 5$) are key to our results.

2.1 Production of human capital

As agents enter parenthood, they are heterogeneous in human capital. One cause of this heterogeneity is exogenous innate ability. Ability differences partly explain income differences in most empirical and theoretical work. Thus it is natural to consider persistence in ability in investigations of income persistence. Persistence of this sort is a feature in both Restuccia and Urrutia (2004) and Holter (2014). Let $\hat{a}$ be the ability of a child and $a$ be the ability of this child’s parent. For each dynasty, the sequence of abilities across generations follows an AR(1) process

$$\ln(\hat{a}) = \rho_a \ln(a) + \epsilon, \epsilon \sim N(0, \sigma_\epsilon^2).$$

(1)

Here $\rho_a$ is the intergenerational correlation of innate abilities and $\epsilon$ is a shock to ability. Ability in each dynasty follows the same process and the shocks are independent across dynasties.\(^2\) There is a high notational cost for precision in our model. We opt to limit this where possible and adopt a shorthand. We do not index dynasties with the understanding that the productivity measure, $a$, and many endogenous items pertain to a dynasty. Also we do not index time. Instead, items with no qualifiers refer to adults and the $\hat{}$ notation refers to their children.

An agent’s human capital accumulates according to

$$\hat{h} = \hat{h}(\hat{a}, h, e_1, e_2, e_3).$$

(2)

Here $h$ is parental human capital and $e_k$ is a measure of education expenditures in the $k^{th}$ period of life where $k \in \{1, 2, 3\}$. Human capital has a genetic component through $\hat{a}$, a socioeconomic component through $h$, and a means to modify these preordained inputs through education expenditures, $e_k$. In our model the three components are reinforcing in that those with more innate ability tend to have parents with more human capital and income. As such on average they receive a socioeconomic advantage as well as more education expenditures in equilibrium.

\(^2\)Following Tauchen (1986), we approximate this process by a discrete Markov process to restrict the number of states for $a$. 

5
Aside from breaking down expenditures across the three periods in childhood \((e_1, e_2, e_3)\), we further allow both government and parents to invest in education. As in Abington and Blankenau (2013), we specify

\[
e_k = \left( f_k^p + g_k^p \right)^{\frac{1}{\eta}}.
\] (3)

Here \(f_k\) and \(g_k\) are parental and government education spending on the child in the \(k^{th}\) period of childhood. We assume inputs to be equally productive but allow them to be imperfectly substitutable where \(\eta \leq 1\) gauges this substitutability.

Variants of equation (2) pervade the literature on human capital accumulation. For example, Becker and Tomes (1986) propose that human capital is formed by education expenditures and natural endowments, which are genetically inherited from parents.\(^3\) Our focus on the timing of inputs to \(\hat{h}\) puts us in a more narrow literature where human capital accumulation has a multi-stage structure and early childhood is specifically modeled as one distinct stage. Following Cunha et al. (2010), we allow human capital at the end of each period, \(\hat{h}_k\), to be determined by human capital carried into this period \(\hat{h}_{k-1}\), parental human capital, \(h\), and current education investment, \(e_k\). Innate ability can be seen as the starting human capital, \(\hat{h}_0 = \hat{a}\). Accordingly, we specify

\[
\hat{h}_1 = \left( \gamma_{11} \hat{a}^{\phi_e} + \gamma_{12} \hat{h}^{\phi_e} + (1 - \gamma_{11} - \gamma_{12}) e_1^{\phi_e} \right)^{\frac{1}{\phi_e}}.
\] (4)

With this specification, the elasticity of substitution between the various inputs is \(\frac{1}{1-\phi_e}\) and the \(\gamma\) parameters gauge their relative importance.

Early childhood human capital \(\hat{h}_1\) becomes an input in the production of mid-childhood human capital, \(\hat{h}_2\). Parental human capital continues to play a role and further investments are made. These combine to create

\[
\hat{h}_2 = \left( \gamma_{21} \hat{h}_1^{\phi_i} + \gamma_{22} \hat{h}^{\phi_i} + (1 - \gamma_{21} - \gamma_{22}) e_2^{\phi_i} \right)^{\frac{1}{\phi_i}}.
\] (5)

At the end of late childhood, the agent has lifetime human capital equal to

\[
\hat{h} = \hat{h}_3 = A \left( \gamma_{31} \hat{h}_2^{\phi_i} + \gamma_{32} \hat{h}^{\phi_i} + (1 - \gamma_{31} - \gamma_{32}) e_3^{\phi_i} \right)^{\frac{1}{\phi_i}}.
\] (6)

This human capital production process allows distinctions between early and later childhood through both weights and elasticities. Note that we keep the elasticity parameter constant through the two periods of later childhood. We opt for this specification as it more succinctly captures the

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\(^3\)See also Becker and Tomes (1979), Su (2004), Restuccia and Urrutia (2004) and Holter (2014).
singular role of early childhood. In this specification we do not separate out time investment. We note however, that a large share of all inputs are time inputs. For example, more than 80 percent of total spending at the primary and secondary levels are for labor costs (Aud et al., 2012). This time input is counted as government expenditures in our model. To the extent that families provide a direct instructional input, our calibration understates their input. However, in large part this parental time input can be considered part of the socioeconomic contribution of the family and thus included in parental human capital.

2.2 Labor market and output

Parents enter the labor market with human capital $h$ in the fourth period. To reflect gains from experience, human capital grows at rate $z - 1$. The value of $z$ exceeds 1 and is common across agents. The productivity per unit of human capital is stochastic. Output is linear in human capital weighted by productivity. This convenience makes the wage rate per unit of human capital exogenous and equal to the productivity measure. Since the wage equals the marginal product of human capital in a competitive labor market, we use the same notation for each.

The gross income of an agent in early adulthood is $hw_4$ where $w_4$ is both the wage per unit of human capital and the productivity scalar. Both $h$ and $w_4$ are agent-specific. The distribution of $w_4$ for any generation $t$ matches the stochastic steady-state distribution of wages deriving from the AR(1) process discussed below. Each agent receives a draw from this distribution in their first working period. Wages in subsequent periods are determined by

$$\ln w_k = \rho_w \ln w_{k-1} + \varepsilon_k, \; \varepsilon_k \sim N(0, \sigma_w^2)$$

for $k \in \{5, 6, \ldots, 10\}$. Here $w_k$ is wage in the $k^{th}$ period of life, $\rho_w \in [0, 1)$ is the intertemporal wage persistence, and $\varepsilon_k$ is the period $k$ shock to wages.\footnote{Again, we use Tauchen (1986)'s method to mimic this process by a discrete Markov process.} The gross income and output of an agent in period $k$, then, is $z^{k-4}hw_k$.

There are seven generations of workers in any period. They differ in human capital, wage, and stage of life. Let $\psi(h, w, k)$ denote the joint probability density function of these state variables. Then output, $Y$, is

$$Y = \sum_k \int_{H \times W} z^{k-4}wh\psi(h, w, k)dhdw$$

for $k \in \{4, 5, \ldots, 10\}$ and $H$ and $W$ representing the range of wages and human capital.
2.3 Agents’ decisions

Children make no economic choices. Parents value consumption in the nine periods of their adulthood and also value the human capital of their offspring. In the first three periods of adulthood, parents make consumption, saving, and education investment decisions to maximize the value of expected utility as described below. When the human capital of their offspring is formed and their children enter adulthood, parents only choose consumption and saving in the remaining periods of life. We now present the maximization problems of parents in different periods of life.

As the first period of adulthood begins, parents know their own human capital and all aspects of government policy. Their current wage and the ability of their offspring are stochastic but revealed prior to decisions. Given the current state \((h, \hat{a}, w_4)\), parents choose current consumption, education spending, and bond holdings to solve the following Bellman problem:

\[
V_4(h, \hat{a}, w_4) = \max_{c_4, f_1, b_4} \left\{ \frac{c_4^2}{\sigma} + \beta E[V_5(h, \hat{h}_1, w_5, b_4)] \right\}
\]

subject to

\[
c_4 + f_1 + b_4 = hw_4(1 - \tau),
\]

\[
e_1 = (f_1^\eta + g_1^\eta)^{\frac{1}{\eta}},
\]

\[
\hat{h}_1 = \left( \gamma_{11} \tilde{a}^{\phi_e} + \gamma_{12} \tilde{h}^{\phi_e} + (1 - \gamma_{11} - \gamma_{12}) e_1^{\phi_e} \right)^{\frac{1}{\phi_e}},
\]

\[
c_4, f_1 \geq 0.
\]

The discount rate is \(\beta < 1\), \(\sigma < 1\) gauges marginal utility, and \(\tau\) is the tax rate on labor income. The period \(k\) value function is given by \(V_k(\cdot)\). Here \(b_4\) measures the number of bonds purchased in period 4. The price of a bond is one unit of the consumption good and each yields \(r\) units of the consumption good in the subsequent period. Parents are constrained by their budget and by the non-negativity of consumption and education spending. They are also subject to the relationship between \(f_1\) and \(\hat{h}_1\) given by equations (3) and (4) with \(k = 1\).

The set of state variables changes through the life cycle. In the second period of adulthood, the state space becomes \((h, \hat{h}_1, w_5, b_4)\). Innate ability, \(\hat{a}\), is subsumed into \(\hat{h}_1\) and parents additionally differ by \(b_4\). The expectations operator is required due to uncertainty in period 4 regarding the wage in following periods.

As a sensitivity analysis, we will consider the case where parents are not allowed to borrow in early adulthood. This borrowing constraint requires additionally that \(b_4 \geq 0\). We do not consider
such constraints beyond early adulthood for three reasons. First, evidence by Caucutt and Lochner (2012) suggests that young parents are significantly more constrained than old parents. Second, as discussed below, private spending on education is highest in early childhood. Government provides most education expenditures in later periods so there is little scope for constraints to restrain private spending. Third, we find that even constraints in early adulthood do not have a large impact on the effectiveness of government spending in reducing persistence. As such, more constraints do not warrant the additional complexity.

Upon learning the wage in the second period of adulthood, \( w_5 \), the agent solves

\[
V_5(h, \hat{h}_1, w_5, b_4) = \max_{c_5, f_2, b_5} \left\{ \frac{c_5^\sigma}{\sigma} + \beta E[V_6(h, \hat{h}_2, w_6, b_5)] \right\}
\]

subject to

\[
c_5 + f_2 + b_5 = zw_5(1 - \tau) + rb_4,
\]

\[
e_2 = (f_2^\eta + g_2^\eta)^{\frac{1}{\eta}},
\]

\[
\hat{h}_2 = \left( \gamma_{21} h_1^{\phi_1} + \gamma_{22} h_2^{\phi_1} + (1 - \gamma_{21} - \gamma_{22}) e_2^{\phi_1}\right)^{\frac{1}{\phi_1}},
\]

\[
c_5, f_2 \geq 0.
\]

The problem is similar to that in the first period of adulthood. There are now four state variables due to differences in bond holdings. The set of state variable for the subsequent period is analogous.

As the third period of adulthood begins, parents make a final education spending choice and \( \hat{h} \) is subsequently determined. For this reason, we include the parent’s valuation of \( \hat{h} \) to period 6 utility and the Bellman problem is

\[
V_6(h, \hat{h}_2, w_6, b_5) = \max_{c_6, f_3, b_6} \left\{ \frac{c_6^\sigma}{\sigma} + \xi \frac{\hat{h}^\sigma}{\sigma} + \beta E[V_7(h, w_7, b_6)] \right\}
\]

subject to

\[
c_6 + f_3 + b_6 = z^2 hw_6(1 - \tau) + rb_6,
\]

\[
e_3 = (f_3^\eta + g_3^\eta)^{\frac{1}{\eta}},
\]

\[
\hat{h} = \hat{h}_3 = A \left( \gamma_{31} \hat{h}_2^{\phi_1} + \gamma_{32} h_2^{\phi_1} + (1 - \gamma_{31} - \gamma_{32}) e_3^{\phi_1}\right)^{\frac{1}{\phi_1}},
\]

\[
c_6, f_3 \geq 0.
\]

The scalar \( \xi \) indicates the relative importance of the child’s accumulated human capital to the parent.
The notation for \( V_7 \) shows one fewer state variable moving forward. Beyond the third period of adulthood, agents can influence future utility only through consumption so the education state of their offspring is dropped. Agents differ only by human capital, savings, and wage. For periods \( 7 - 10 \) the problem is

\[
V_k(h, w, b_{k-1}) = \max_{c_k, b_k} \left\{ \frac{c_k}{\sigma} + \beta E[V_{k+1}(h, w_{k+1}, b_k)] \right\}, \ k \in \{7, 8, \ldots 10\}
\]

subject to \( c_k \geq 0 \) and \( c_k + b_k = z^{k-4}hw_k(1 - \tau) + rb_{k-1} \). Due to retirement, human capital and wage are not relevant in the final two periods. Furthermore, an agent in the final period consumes the proceeds of prior savings so

\[
V_{11}(b_{10}) = \max_{c_{11}, b_{11}} \left\{ \frac{c_{11}}{\sigma} + \beta E[V_{12}(b_{11})] \right\},
\]

\[
V_{12}(b_{11}) = \frac{c_{12}}{\sigma} = \frac{(rb_{11})^\sigma}{\sigma}.
\]

### 2.4 Government

Government taxes labor income at rate \( \tau \), collecting revenue equal to \( \tau Y \). Revenue is allocated across education expenditures for the unit mass of agents currently in early childhood and the distinct unit masses of agents currently in each of middle and late childhood. Government must balance its budget in each given period. Then, the government budget constraint is

\[
ge_e + g_m + g_l = \tau Y
\]

where \( g_e, g_m, \) and \( g_l \) are total current government spending in early, middle, and late childhood. In most of our investigation, government spends the same amount on each student. With a unit mass of children in each education stage, government spending \( g_e, g_m \) and \( g_l \) are both total and per pupil spending levels. That is \( g_e = g_1, g_m = g_2, \) and \( g_l = g_3 \). We also consider the case where government spends more in early childhood on agents in the lowest income quintile than on those with higher incomes. In this case, we define

\[
ge_e = 0.2g_{1,l} + 0.8g_{1,h}
\]

where \( g_{1,l} \) is per capita spending on lower income students and \( g_{1,h} \) is per capita spending on higher income students.

In the United States, education spending by local government is quite unequal across districts. This aspect of public funding to some extent allows parents to choose a level of government spending
through Tiebout sorting. For this reason, researchers sometimes choose to count local government expenditures as private expenditures. However, we opt to count local spending as government spending for four reasons. First, our human capital production function makes family and government inputs imperfectly substitutable. Local, state, and federal funding are, by and large, components of a single budget for a district; i.e. they are close to perfect substitutes. Direct family spending, in contrast, buys a separate set of inputs which are more plausibly imperfect substitutes. Second, spending on education by choosing location often is intended to provide students with socioeconomic advantages. Since we have included parental human capital to account for such advantages, including expenditures for this purpose is less important. Third, the complex funding and redistribution formulas governing state support for education weaken the link between local taxation and local spending. Finally, education spending is only one of many concerns in locating the family. It is difficult to disentangle the education and consumption motives in choosing a location.

The value $\tau$ measures not only the tax rate but also the share of output allocated to government education spending. We investigate the effects of tax level $\tau$ and the allocation of tax across early and mid/late childhood. Thus we define

$$\zeta_e = \frac{\alpha_e}{\tau}, \quad \zeta_m = \frac{\alpha_m}{\tau}, \quad \zeta_l = \frac{\alpha_l}{\tau}$$

such that $\zeta_e$, $\zeta_m$, and $\zeta_l$ are the shares of output allocated by government to early, middle, and late childhood.

### 2.5 Equilibrium definition

We do not explicitly model the investment side of the economy. Instead we assume that the bond market is international and accommodates any shortages and surpluses in the goods market. With this we are able to define an equilibrium in this setting. Let $\zeta_k$ denote the state space for an agent in the $k^{th}$ period of life.

A stationary recursive competitive equilibrium is a set of functions $f_{k-3}(\zeta_k)$ for $k \in \{4, 5, 6\}$, $V_k(\zeta_k)$, $c_k(\zeta_k)$, $b_k(\zeta_k)$ for $k \in \{4, 5, \ldots, 11\}$, $V_{12}(\zeta_{12})$, $c_{12}(\zeta_{12})$ and the aggregate quantity $Y$ such that: 1) given government spending and tax rules, agents’ decisions are optimal; 2) the wage per unit of human capital for each agents is equal to productivity so that the labor market clears; 3) the government sets $g_e$, $g_m$, $g_l$, and $\tau$ to satisfy equation (9) with output given in equation (8); 4) surpluses and shortages in the goods market are accommodated by the international bond market;
Table 1: Parameters set exogenously.

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intergenerational persistence of innate ability</td>
<td>$\rho_a$</td>
<td>0.25</td>
</tr>
<tr>
<td>Intertemporal wage persistence for young (older) workers</td>
<td>$\rho_w$</td>
<td>0.46 (1)</td>
</tr>
<tr>
<td>Discount rate</td>
<td>$\beta$</td>
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<tr>
<td>Intertemporal preference parameter</td>
<td>$\sigma$</td>
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<tr>
<td>Interest rate</td>
<td>$r$</td>
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<tr>
<td>Substitutability parameter in early childhood</td>
<td>$\phi_e$</td>
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<tr>
<td>Substitutability parameter in middle and late childhood</td>
<td>$\phi_l$</td>
<td>-0.78</td>
</tr>
<tr>
<td>Share of GDP on early childhood education by government</td>
<td>$\zeta_e$</td>
<td>0.004</td>
</tr>
<tr>
<td>Share of GDP on middle childhood education by government</td>
<td>$\zeta_m$</td>
<td>0.017</td>
</tr>
<tr>
<td>Share of GDP on late childhood education per period by government</td>
<td>$\zeta_l$</td>
<td>0.019</td>
</tr>
</tbody>
</table>

5) the optimal decisions and evolution of exogenous states result in a time invariant distribution of agents across states.

3 Calibration

We calibrate parameter values to form a baseline economy that matches the U.S. data. When available, we use empirical counterparts to the parameters of our model. In other cases, the parameters are simultaneously calibrated so that features of the generated data match features of the U.S. economy. Table 1 shows our choice of parameters set exogenously.

The first two parameters govern the persistence of shocks. Our specification of $\rho_a$ is meant to capture only the genetic aspects of ability transference. Since direct observations of ability are not available, researchers consider proxies. For example, Black et al. (2008) show that the intergenerational persistence of IQ scores is 0.32. However, the IQ measure is a combination of innate ability (nature) and malleable ability (nurture). For our purposes, this measure may overstate the persistence of ability. Restuccia and Urrutia (2004) and Holter (2014) calibrate $\rho_a$ to match earnings persistence. They choose $\rho_a = 0.2$ and $\rho_a = 0.332$, respectively. We instead fix this parameter by choosing a mid point, $\rho_a = 0.25$, and match persistence through other parameters as discussed below. Our choice of $\rho_w$ is from Karahan and Ozkan (2012). They find that the persistence of earnings is relatively low early in a career and increases to near unity midway through one’s working life. Specifically, they report persistence of 0.878 before the age of 33 and nearly 1 later in life. Based on these estimates and adjusting for the length of periods in our model, we set $\rho_w = 0.46$ for workers in the first three working periods and $\rho_w = 1$ thereafter.
For preference parameters we set the discount factor $\beta$ to 0.78 and the elasticity parameter $\sigma$ to $-0.5$. The discount rate corresponds to the commonly used annual rate of 0.96. The estimate of elasticity parameter $\sigma$ ranges between $-2$ and $0.5$ in the empirical literature.\(^5\) We choose a midpoint value $\sigma = -0.5$. Our choice of $r$ corresponds to a risk free annual interest rate of 1.011. This is calculated by Holter (2014) using data of 3-month T-bill rates over the period 1947-2008.

The next two parameters are central to our analysis. The substitutability parameters $\phi_e$ and $\phi_l$ gauge the interactions among the inputs in human capital production. They largely determine the roles of education investment at different stages. Cunha et al. (2010) estimate a multi-stage production function with both cognitive and non-cognitive skills. As a special case, they consider a model with only cognitive skills. This special case is the best available analogy in the literature to the environment we are modelling. We use their multi-stage production function and features of its calibration. In their specification, inputs in each period are current period cognitive skills, parental cognitive skills, and current period investments. These are closely related to our measure of human capital at the beginning of the period, parental human capital, and current expenditures. As such, we take their elasticity measures and set $\phi_e = -0.18$ and $\phi_l = -0.78$. With these estimates, items in both early and mid/late childhood are more complementary than in the Cobb-Douglas case and later childhood complementarity exceeds that in early childhood. This means that low investment early in life cannot easily be compensated for by increased later spending.

The policy parameters $\zeta_e$, $\zeta_m$ and $\zeta_l$ represent public expenditures on early, middle, and late childhood as a percentage of GDP. Heymann et al. (2004) report and compare government expenditures on early childhood education and care by the United States and its European peers. They find that nearly 0.4 percent of U.S. GDP is spent by the government on early education.\(^6\) Accordingly, we set $\zeta_e = 0.004$. Education at a Glance (2007) shows that public expenditures on primary education, lower secondary, and upper secondary education represent 1.7, 0.97, and 0.93 percent of GDP. The six years of primary and secondary education are parallel to middle and late childhood in our model, so we set $\zeta_m = 0.017$ and $\zeta_l = 0.019$.

With these parameters set exogenously, we next set the remaining nine parameters to match eight relevant moments generated by our model with their empirical counterparts. We balance the government’s budget constraint as a ninth condition. The first column of Table 2 lists the nine

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\(^5\)See Keane and Wolpin (2001) and Hubbard et al. (1994).

\(^6\)Data from UNESCO/OECD (2003) shows that private and public expenditures on educational institutions count for 0.4 percent of GDP and less than 10 percent of it is private expenditures.
parameters to be estimated with the corresponding targeted moments indented below. In general, each parameter has some effect on each of the moments but is relatively closely associated with one moment. The second and third columns give the notation and chosen value for the parameter. The final two columns give the empirical observation of the moment and its counterpart generated by the model.

The first row shows that we use the weight of children’s human capital, $\xi$, to target the intergenerational persistence of earnings. Based on the empirical work of Grawe (2004), we set our target to be $\rho = 0.47$. Intergenerational persistence of earnings has three sources in our model. First, a child’s innate ability is related to parental ability through equation (1). Since ability is an argument in human capital accumulation, the parameter $\rho_a$ governs persistence through this channel. Second, there is a socioeconomic effect. Roemer (2004) points out that parents can positively influence children through a family culture that values skills and discipline, and through social connections that facilitate access to opportunities. In our model, this is captured by the inclusion of parental human capital in the human capital production function and is governed by $\gamma_{12}$, $\gamma_{22}$, and $\gamma_{32}$. Finally, family education expenditures depend on family income. Through this channel, income is passed across generations through formal education. The strength of this relationship depends on $\xi$ as it gauges the importance of children’s human capital to parents.

Any of these parameters might be used to target persistence. Since empirical estimates of $\rho_{a}$ are in a sufficiently tight range, we take this value as better determined outside the model and set it as discussed above. The other two channels are both estimated endogenously. We find it most convenient to target $\rho$ using $\xi$. In the end, however, our choices of the $\gamma$ values also have influence and are consistent with this degree of persistence.

As mentioned above, the functional form for our human capital accumulation is motivated by the work of Cunha et al. (2010). We use their estimates of elasticities directly, which is one motivation for adopting this specification. However, their estimates of weights ($\gamma$ values) depend on the scaling of inputs and do not directly apply. Instead, we calibrate these. To do this we make several assumptions which allow a more intuitive and parsimonious calibration.

To better see the intuition behind our calibration, it is useful to look at a simplified version of the model. The recursive nature of our human capital production function obscures the interpretation of the $\gamma$ parameters. For example, the weight on ability in the production of $\hat{h}_1$ is $\gamma_{11}$. However, our concern is with the importance of ability in generating $\hat{h}_3$ since this is the determinant of income.
Table 2: Parameters set endogenously. The indented item below the parameter description shows the item targeted by the parameter. The first data column shows the value chosen for the parameter. The last two columns compare the value of the targeted item in the data with that generated by the model. In the case of the budget balance we target 0 (balanced budget) rather than the observed budget balance.
Early childhood human capital, \( \hat{h}_1 \), is only one input in the production of \( \hat{h}_2 \) which in turn is only one input in the production of \( \hat{h}_3 \). As such, the weight on ability in producing \( \hat{h}_3 \) is much smaller than \( \gamma_{11} \).

For a simpler interpretation, we consider the special case where \( \phi_e = \phi_l = \phi \). With this simplification, equations (4)-(6) reduce to

\[
\hat{h} = \left( \gamma_1 \alpha + \gamma_2 \phi + \gamma_3 \epsilon_1 + \gamma_4 \epsilon_2 + \gamma_5 \epsilon_3 \right)^{\frac{1}{2}}.
\]

(11)

Here \( \gamma_1 \) through \( \gamma_5 \) sum to 1 and are functions of the underlying \( \gamma \) parameters in equations (4)-(6).\(^7\) These coefficients gauge the relative importance of different inputs in the production of human capital. Since \( \phi_e \neq \phi_l \), the true weight on ability, for example, differs slightly from \( \gamma_1 \). However, considering \( \gamma_1 \) to be the relative importance of ability is without consequence in our results and allows a more succinct discussion.

We proceed by calibrating the \( \gamma \) values in equation (11) and mapping them into the \( \gamma \) values in the more general case. Two reasonable assumptions are imposed for simplification. In equation (11) the total weight on parental human capital is given by \( \gamma_2 = \gamma_{12} \gamma_{21} \gamma_{31} + \gamma_{22} \gamma_{31} + \gamma_{32} \). The three right-hand side elements reflect the contribution of parental human capital at each stage. Our first assumption is that parental human capital in each period is equally important which leads to \( \gamma_{12} \gamma_{21} \gamma_{31} = \gamma_{22} \gamma_{31} = \gamma_{32} \). Our second assumption is that expenditures in middle and late childhood are equally important while early childhood expenditure may be more productive.\(^8\) This gives \( \gamma_3 = \kappa \gamma_4 = \kappa \gamma_5 \) where \( \kappa \) is a constant and expected to exceed one. With these assumptions and the fact that all \( \gamma \) parameters in equation (11) add up to 1, we can recover estimates of all \( \gamma \) parameters in (4)-(6) from knowledge of \( \gamma_1, \gamma_4, \) and \( \kappa \). We reiterate that these assumptions are made to clarify and simplify the calibration. They are not imposed in solving the model.

We choose these three parameters along with the substitutability parameter \( \eta \) to match four features of parental expenditures on education. Two features relate to the level of spending and two to the distribution of spending. While we do target one parameter to each of the four moments as a technical matter, we find them to be closely related and discuss them as jointly determined. The close relationship is natural given the disparate influences on spending. In general, a family will spend more on a stage of education when income is higher, when government spending at that stage is lower, and when spending is more productive at that stage. Also families will spend more

\(^7\) Note that \( \gamma_1 = \gamma_{11} \gamma_{21} \gamma_{31}, \gamma_2 = \gamma_{12} \gamma_{21} \gamma_{31} + \gamma_{22} \gamma_{31} + \gamma_{32}, \gamma_3 = \gamma_{13} \gamma_{21} \gamma_{31}, \gamma_4 = \gamma_{23} \gamma_{31}, \) and \( \gamma_5 = \gamma_{33}. \)

\(^8\) The higher productivity of early investment is documented in Cunha et al. (2005) and Knudsen et al. (2006).
when government spending is a less perfect substitute for family spending.

The moments related to the level of spending are family spending in early and later childhood as a share of total spending in these periods. Barnett and Masse (2003) estimate that about 60 percent of early childhood spending is private so we target \( \frac{f_1}{g_1 + f_1} = 0.6 \). Using data from Education at a Glance (2007), we find that expenditures on primary and secondary education by households represent 8.6 percent of total expenditures from all sources. Therefore we target \( \frac{f_2 + f_3}{g_2 + g_3 + f_2 + f_3} = 0.086 \). The moments related to the distribution of spending come from Expenditures on Children by Families (2011). This study finds that the ratio of early education spending by parents in the middle income tercile to that of parents in the highest tercile is 0.556. For later childhood spending, this ratio drops to 0.33. Our model generates a slightly lower ratio in early childhood than its data counterpart. This is partly because parents spend on children only for the return of education in our model. However, in actual economies, some expenses on childcare are for children’s basic needs and thus are not avoidable even among the poor.

Each of the four moments is relatively sensitive to the four parameters under consideration. For example, later spending by families as a share of the total depends upon its productivity, \( \gamma_4 \) and \( \gamma_5 \), directly. It is also related to the relative weight of early childhood expenditure, \( \kappa \), since desired spending can be allocated to any period. Thus the productivity of first period education influences second and third period spending. The spending ratios discussed above are related closely to the substitutability between family and government expenditure, \( \eta \). A lower value of \( \eta \) increases the ratio of spending by middle income and high income families in both early and later childhood. To see why, consider the case where \( \eta = 1 \). With this perfect substitutability, government spending can fully crowd out spending for the middle income families while high income families continue to spend. In this case the ratio is zero. At lower values of \( \eta \), middle income families spend positive amounts and the ratio exceeds zero. The ratio continues to increase as \( \eta \) approaches negative infinity. With increased complementarity, there is a tendency to bring private spending closer to public spending. The observed drop-off in the spending ratio from 0.556 to 0.33 between early and later childhood is due in large part to higher government spending at the later stage. The relative size of these ratios, then, provides information about how much private spending responds to public spending. This is again influenced by \( \eta \). Our estimate of \( \eta \) is 0.48, indicating that parental and government expenditures are not perfect substitutes.9

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9Studies that also specify private and public inputs as imperfect substitutes include Arcalean and Schiopu (2010),
Variable Baseline No public spending \( \varsigma_e = 0 \) \( \varsigma_m = \varsigma_I = 0 \)

<table>
<thead>
<tr>
<th>Persistence of earnings</th>
<th>0.469</th>
<th>0.667</th>
<th>0.552</th>
<th>0.623</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variance of (log) earnings</td>
<td>0.361</td>
<td>0.382</td>
<td>0.390</td>
<td>0.359</td>
</tr>
</tbody>
</table>

Table 3: Effects of current policy. The first data column shows results using the baseline government education spending at all stages of education. The second column shows results when spending at all levels is set to 0. In the third and forth columns spending in early childhood and then later childhood is set to 0 with the other set at its baseline level.

We choose the standard deviation of wage shocks, \( \sigma_w \), to target the variance of log earnings of 0.36. We take this value from Mulligan (1997) who estimates the variance of permanent income of all workers using the U.S. data. Storesletten et al. (2004) find that about 60 percent of the variation in earnings is accounted by worker characteristics prior to labor market entry.\(^{10}\) In our model, human capital is determined when agents enter the labor market, so we target the variance of log human capital of 0.24 with the standard deviation of ability shocks, \( \sigma_a \). We choose the experience parameter, \( z \), to match the growth of earnings over the life cycle. We take the Heckman et al. (2006) estimates from a Mincer earnings regression and calculate that earnings increase by 55.2 percent with 42 years of working experience. Our choice of \( z \) matches this moment precisely.

We choose \( A \) to balance the government budget. From equations (8) and (9) the budget constraint can be written as

\[
\tau = \frac{g_e + g_m + g_l}{\sum_k \int_{H \times W} \int_{k} \psi(h, w, k) dh dw}
\]

Using equation (10), we calibrate \( \tau \) exogenously as \( \varsigma_e + \varsigma_m + \varsigma_I \). Equation (12), then, will hold only for the proper relationship between government education expenditures and human capital. The scalar parameter \( A \) governs this relationship. Thus we choose \( A \) to scale human capital and output such that this relationship is satisfied.

4 Current policy

To set the stage for our policy experiments, we first demonstrate the impact of current policy. We compare the economy in the case where government spends at current levels to the case where it does not spend at all \( (\varsigma_e = \varsigma_m = \varsigma_I = 0) \). We then consider shutting down spending in either early or later education. The results are reported in Table 3.

\(^{10}\) This finding is consistent with other studies in the literature. See Zimmerman (1992) and Huggett et al. (2011).
Comparing the first and second columns, we see that government spending has a substantial effect on persistence. Spending at current levels yields intergenerational income persistence equal to 0.469. Absent government intervention in education, this would increase to 0.667. Each stage of education plays a role in reducing persistence. The third column shows that shutting down only early education spending would increase persistence to 0.552 while shutting down only later spending would increase persistence to 0.623. The effect of shutting down later spending is nearly twice as large as the change from shutting down early spending. However, total government spending on later childhood is nine times as large as in early childhood. Thus the current level of early childhood spending has a larger effect on persistence per unit of expenditure.

Figure 1 helps to explain how government spending in early childhood reduces persistence. In the first panel, the solid curve represents the Lorenz curve for private spending on early education in the economy with no government spending. The dashed line further to the right shows the Lorenz curve for private spending when government spends at its current level. When government spends, crowding out of private expenditures occurs. The common level of government spending is larger relative to income for those in the lower end of the income distribution. This induces a more pronounced crowding out effect for these families. As a consequence, private education spending becomes more unequal when public spending increases. For example, without government spending the lower spending tercile accounts for 13.7 percent of all family spending. When government spends at current levels, this drops to 9.6 percent.

While private spending becomes less equal with government spending, total spending at this stage \((f_1 + g_1)\) becomes more equal. The Lorenz curves for total spending with and without government spending are shown in the second panel of Figure 1. The share of total spending by the lower spending tercile increases from 13.7 to 19.6 percent. This change in the distribution of education spending weakens the link between family income and early childhood expenditures. Consequently it lowers the intergenerational persistence of income.

Figure 2 considers the same exercise for later childhood spending. The first panel shows a more dramatic rightward shift of the Lorenz curve for private spending as a consequence of government spending. The second panel shows that aggregate spending, in contrast, is much more equally distributed. Since \(g_2\) and \(g_3\) are common to all agents and account for over 90 percent of all spending in later childhood, there is relative equality of total spending on later childhood education. Current levels of spending considerably weaken the income/expenditure link for later childhood.
Figure 1: The solid curve in Panel 1 is the Lorenz curve of private spending in early childhood with no government spending. The dashed curve is the Lorenz curve of private spending in early childhood under current government spending levels. Panel 2 shows the same Lorenz curves for total spending.
Figure 2: The solid curve in Panel 1 is the Lorenz curve of private spending in later childhood with no government spending. The dashed curve is the Lorenz curve of private spending in later childhood under current government spending levels. Panel 2 shows the same Lorenz curves for total spending.

Little inequality remains and there is little scope for further improvements through this channel.

While our focus is on the persistence of income across generations, government spending also influences the distribution of income within a period. The second row of Table 3 shows that absent government spending on education, the cross-sectional variance in income would increase by about 6 percent to 0.382. The third and fourth columns demonstrate some of the competing effects of spending on the variance of income. Eliminating only early childhood spending increases the variance of income by more than eliminating both. Eliminating only later spending, in contrast, lowers this variance. When government spends more at a level, spending becomes more equal. This serves to lower the variance of income. Government spending also serves to increase the level of spending in aggregate. Increased spending serves to amplify ability differences and increase the variance of income. For early childhood, family spending is a large part of the total spending. The first effect dominates and more equal spending results in more equal income. For later childhood, government spending is large relative to the total. The high level of spending affects those with higher ability disproportionately and income variance is larger. Any change in government spending
will have these two contrasting effects. In general, the overall effect on the variance of income within a generation is small.

5 Policy experiments

We establish above that government education funding at current levels has a large effect on the persistence of income across generations. We now consider the extent to which further increases in spending can yield further reductions. To identify a reasonable experiment, we look at other developed economies for feasible policies. Denmark and Norway have the lowest earnings persistence (0.15 and 0.17) among all the OECD countries. They also allocate the largest share of GDP (0.8 percent) to early education. Our first experiment considers the extent to which the difference in early childhood funding contributes to the difference in persistence between the U.S. and these countries. To this end, we consider what would happen if early childhood spending in the U.S. were increased to match the higher rate seen in these countries. That is, we consider the effect of increasing $\zeta_e$ by 0.4 percent of GDP to $\zeta_e = 0.008$. In the second experiment, expenditure is increased by the same amount but allocated only to the lowest income quintile. This is motivated by our observation that changes in persistence are in large part due to changes in human capital at the lower end of the income distribution.

We next consider whether later childhood spending provides another avenue for reducing persistence. We consider a policy of increasing spending in later childhood to match that of Denmark and Norway. These countries allocate 4.1 percent of GDP to later childhood education. To match this, we increase later childhood spending by 0.5 percent of GDP, allocating this equally across the two periods of later childhood. That is, we set $\zeta_m = 0.0195$ and $\zeta_l = 0.0215$.

In the next two experiments, we consider the extent of the opportunity to further reduce persistence through public spending on early childhood. We explore the range of possible reductions by increasing early spending to 1.5 and then 2 times the level in Denmark. We demonstrate diminishing returns in terms of reduction in persistence. In the final experiment we consider a reallocation of current expenditures. Since forgone later spending has a smaller effect on persistence than the positive increment to early childhood spending, a reallocation can yield reduced persistence with unchanged levels of spending. To demonstrate, we adjust spending levels so that the current level of government education spending is allocated equally across its three uses. This requires $\zeta_e = \zeta_m = \zeta_l = 0.013$. 

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Table 4: Impact of policy changes. The first data column reiterates the baseline case. In the second and third columns government spending on early childhood is increased for all agents (Early) and for the lowest income quintile (Early*). In the fourth column government spending on later childhood is increased. In columns five and six, larger increases in early childhood spending are considered. In the final column the current level of government spending is reallocated such that government spending in each period of childhood is equivalent.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Baseline</th>
<th>+0.4% of GDP</th>
<th>+0.5% of GDP</th>
<th>+0.8%</th>
<th>+1.2%</th>
<th>Reallocation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Persistence</td>
<td>0.469</td>
<td>0.444</td>
<td>0.391</td>
<td>0.466</td>
<td>0.423</td>
<td>0.407</td>
</tr>
<tr>
<td>Output</td>
<td>1</td>
<td>1.106</td>
<td>1.047</td>
<td>1.026</td>
<td>1.178</td>
<td>1.238</td>
</tr>
</tbody>
</table>

5.1 Results

Table 4 shows the impact of these policy changes. The first row gives our key findings regarding persistence. The second data column (Early) shows that setting U.S. spending levels on early childhood equal to those of Denmark and Norway decreases persistence from 0.469 to 0.444. The decrease in persistence of 0.025 represents 8.3 (7.8) percent of the observed difference between the U.S. and Norway (Denmark). Of course these economies differ along many dimensions. A wide range of circumstances and policies may play a part in determining the persistence differential. Our results show that early childhood spending plays a modest role in understanding this gap. The magnitudes are in the line with the study by Holter (2014). He introduces Danish public education policy by spending more in secondary and tertiary education and less in primary education. He finds that the gap between the U.S. and Denmark is reduced by 9.6 percent. We find an effect nearly 80 percent as large by focusing only on early childhood education.

The third column (Early*) considers the same 0.4 percent increase in early childhood. Now, however, this increment is allocated fully to the offspring of the lowest income quintile. Rather than reducing persistence from 0.469 to 0.444, persistence is reduced to 0.391; the change is three times as large for the same expenditure. This result is consistent with Herrington (2013). He adopts the progressive spending system in compulsory education from Norway and shows that the differences in the distribution of public spending explains about 11 percent of the gap. Our estimates suggest that spending the same level on early education as Norway together with a focus on the poor group eliminates 26 percent of the gap in persistence. Our result is also related to Erosa and Koreshkova (2007). They build a dynastic model of human capital to examine the effects of progressive taxation. Their results show that replacing the current progressive income tax system with a proportional
one increases earnings persistence. Both papers show that policy favoring low income families is more effective in lowering persistence.

The fourth column shows that an even larger increase in spending has almost no effect on persistence when allocated to the later stages of education. Persistence falls only from 0.469 to 0.466. Understanding this result clarifies the perils of reducing persistence through government education spending. As with the discussion above regarding variance, the result hinges on conflicting effects of government funding. Weakening the link between family spending on education and family income is the key to reducing persistence through the education channel. But spending has a competing effect on persistence by increasing overall spending levels. The same increase in spending levels has a larger effect on higher ability students who tend to have wealthier parents. At current levels of $\zeta_m$ and $\zeta_l$ the income/expenditure link is already very weak and more spending does little to decrease it further. This channel for spending to influence persistence is nearly shut down. The other continues to operate in the opposite direction and at current levels of spending they nearly offset. Moving beyond this level of spending can lead even to increased persistence.

This insight further illuminates the results regarding progressive spending. With progressive spending, the poor will have considerably higher expenditures on their behalf, weakening the link between spending and income. At the same time, those in the upper end of the income distribution, who tend to have the more able offspring, will not have additional expenditures. This offsetting effect will not operate. Progressive spending is effective because it enhances the first effect and minimizes the second.

While room exists for decreasing persistence through early childhood spending, the fourth and fifth columns show that returns along these lines are diminishing. Upon increasing early childhood spending by an additional 0.4 percent of GDP, we get an additional reduction in persistence. However, while the drop-off in persistence was 0.025 for the initial increment the drop-off is only 0.021 for the second increment and 0.016 for the third. With spending at this stage becoming more equal, the opportunity for further reductions falls and the ability of the more able to profit from increased spending becomes a larger part of the effect on persistence.

As a final experiment we consider a reallocation of current expenditures. The first and third experiments make it clear that at current levels of expenditures persistence moves more in response to early childhood spending than later spending. In the U.S., spending at later stages is much larger than spending on early childhood. This suggests that a reallocation of these expenditures
might have consequences for persistence with no additional cost. The final column of Table 4 confirms this. In this experiment each of the three stages of education receives one third of the total education budget. The result is a decrease in persistence to 0.431, an effect larger than that in our first experiment. This is because reallocation has a larger effect on ς_e, increasing it to 0.13 rather than 0.08. The required decrease in ς_m and ς_l does not induce a negative effect large enough to offset this.

The second row of Table 4 shows the impact of these policies on output. Measures of output are normalized by the level of output in the baseline case. Again expenditures on early childhood education have a bigger effect and again the marginal effect diminishes as expenditures increase. Throughout the range considered, however, the effect on output is large. An increase in early childhood spending of 0.4 percent of GDP results in a nearly 11 percent increase in output. This is partly because government inputs are scarce at this stage and thus productive at the margin. This result is also due to the imperfect substitutability of family and government inputs. Output further increases because of the increase in total education expenditures induced by government involvement in funding education. In the fourth column we see that progressive spending has less than half the effect of across-the-board spending. Progressive spending is relatively efficient for reducing persistence but relatively inefficient for increasing output. This inefficiency reflects that the less skilled cannot transform resources to human capital as readily as the more skilled agents. In this sense progressive spending directs government expenditures toward a less efficient use.

5.2 Driving factors

To this point, we have established that government spending on early childhood is more effective in reducing intergenerational persistence than government spending on later childhood. Our explanation has focused largely on differences in the level of government funding across these two stages. However, our calibration assigns early education two other distinct features. First, early childhood expenditures are relatively more productive in generating human capital than later education expenditures. This is expressed by the relative weight of early expenditures, κ = 2.14. If all expenditures were equally productive we would have κ = 1. Secondly, education inputs are less substitutable in later childhood than are they in early childhood. This is expressed by the substitutability parameters, φ_e = −0.18 and φ_l = −0.78. If later inputs were as substitutable as early inputs, we would have φ_l = −0.18.
Table 5: Equivalent productivity and substitutability. The first row reiterates the baseline parameterization and results. In the second row education inputs in early and later childhood are equally substitutable. In the third row, inputs are equally productive. In the fourth row inputs are both equally substitutable and equally productive.

In this subsection, we evaluate whether these modeling distinctions are important for our results. To do this, we investigate the extent to which spending on early childhood influences persistence when we remove these distinctions. Table 5 reports the results.

To facilitate the comparison, the first row of Table 5 reiterates the results of increasing early childhood spending under our baseline calibration. In the second row, we consider the case where later childhood expenditures are as substitutable with other inputs as early expenditures are. With $\phi_l = -0.18$ the current funding level yields lower persistence. Since early childhood human capital is an argument in the production of later childhood human capital, greater substitutability decreases the adverse effects on persistence of low spending in early childhood. This low spending in early childhood, and the resulting low early childhood human capital, is more effectively compensated for by the higher spending in later childhood. Persistence is lower as a result. The next column shows that with this increased substitutability, policy is also more effective in lowering persistence. Rather than decreasing persistence by 0.025, the same policy change would decrease persistence by 0.034. If early childhood human capital substitutes easily for later spending, early spending by government crowds out later spending by families. This adjustment further weakens the income/expenditure link and reduces persistence.

In the subsequent line, the elasticity measure is returned to its baseline level but the productivity of spending at this stage equals that in later childhood. This lowers intergenerational persistence a bit and dampens the impact of additional expenditures. With these parameters, increased government spending would decrease persistence by just 0.022. This smaller effect reflects that with expenditures less productive, again the scope for mitigating persistence through this channel is diminished.

The fourth row shows the effect of policy when both the elasticity and productivity measures for
Parameters | Baseline economy | +0.4% GDP | Change
--- | --- | --- | ---
$\eta = 0.48$ | 0.469 | 0.444 | $-0.025$
$\eta = 0.9$ | 0.509 | 0.458 | $-0.051$
$\eta = 0$ | 0.182 | 0.202 | $+0.020$
Borrowing constraint | 0.467 | 0.442 | $-0.025$

Table 6: Sensitivity check. The first row repeats results (persistence before and after the policy change) from the baseline case. The second and third rows consider the cases where family and government spending are more and then less substitutable. In the final row borrowing is not allowed for agents in their first period of work.

early childhood are set equal to those in later childhood. In this case, the policy reduces persistence by 0.029. The two changes work counter to each other in gauging the ability of policy to reduce persistence. In aggregate policy is more effective than in the baseline case, but the difference is not large. Throughout policy experiments, increased expenditures in early childhood are more effective than increased expenditures in later childhood. This result is largely due to a current low level of spending in early childhood. The higher productivity of early expenditures amplifies this policy effectiveness while the lower substitutability of these expenditures dampens the effectiveness.

5.3 Sensitivity checks

A crucial parameter in our model is the substitutability between government and private spending. Crowding out of private spending by government spending is a key to our results. When these inputs are more substitutable, we should expect our results to be more pronounced, and vice versa. The centrality of this parameter to our results, along with the absence of relevant estimates in the literature, motivates us to calibrate this parameter endogenously. As noted above, this yields $\eta = 0.48$ so that family and government spending are close but imperfect substitutes. The result reinforces intuition. Still, given the importance of the parameter, we consider policy effectiveness under alternative values for the parameter.

Table 6 gives the results of this sensitivity analysis. Increased substitutability between private and government spending corresponds to increased effectiveness of early childhood spending in reducing persistence. With $\eta = 0.9$ an increase in public spending equal to 0.4 percent of GDP decrease persistence by 0.051, compared to 0.025 in the baseline case. Moving in the other direction, with $\eta = 0$ an increase in government spending yields an increase rather than a decrease in persistence. The explanation is tied closely to the effects of crowding out of private expen-
ditures. When inputs are substitutable, an increase in government spending causes families to decrease spending significantly. For wealthier families, government spending will decrease private spending nearly one-for-one. Lower income families will find government spending to exceed what they would spend in the absence of government spending so a one-for-one crowding out cannot occur. Government spending, then, causes relatively more crowding out among wealthier families, yielding less variability in spending in aggregate.

With inputs less substitutable, a different effect plays a more prominent role. In all cases other than perfect substitutability, government spending increases the productivity of any level of private spending. Since wealthier families spend more, they benefit more from this effect. This effect of policy ties the human capital of the child more closely to that of the parent when government spends more. At \( \eta = 0.48 \), this effect is relatively weak and increased spending lowers persistence. At \( \eta = 0 \), this effect dominates and government spending increases persistence. Note also that the level of persistence is much lower with \( \eta = 0 \). As discussed in the calibration section, smaller values of \( \eta \) yield smaller ratios of education spending across agents at different income levels. This results in less persistence in an otherwise similar economy.

As a final sensitivity analysis, we consider the effect of policy when families are unable to borrow as young adults. This requires \( b_4 \geq 0 \). As shown in the final row of Table 6, borrowing constraints have little effect on persistence. With this constraint imposed, persistence drops from 0.469 to 0.467. Constraints have almost no impact on the effectiveness of policy in reducing persistence. Both with borrowing constraints and without, increasing spending in early childhood by 0.4 percent of GDP reduces persistence by 0.025.

This is not to say that borrowing constraints do not matter. With our baseline calibration, about 80 percent of young parents borrow. When agents cannot smooth consumption through borrowing, they spend less on early childhood education than they otherwise would. The human capital of their offspring decreases and this results ultimately in a 4.4 percent decrease in aggregate output.

While borrowing constraints lower income, they do not importantly change the relationship between income and education expenditure. As such they do not importantly change persistence. In our model agents are prone to borrow because education expenditures occur early in life while income is on average higher later in life. Beyond this, agents with an early adulthood wage shock below the mean are nudged toward greater borrowing with the expectation of better future shocks.
Agents with a wage shock above the mean are nudged toward saving with the expectation of worse future shocks. These motivations for borrowing are common across all income levels. Borrowing constraints limit the early childhood spending of both the wealthy and the poor. With or without constraints, the wealthy spend more on their children and this channel of persistence arises.

6 Conclusion

In the United States, there is a strong correlation between the income of parents and their offspring. With intergenerational persistence of income equal to about 0.47, the mobility of economic status across generations is lower in the U.S. than in most other OECD countries.\textsuperscript{11} Education is one source of persistence. Wealthier families provide a better education to their offspring. This results in more human capital and a higher wage. Public education can lower persistence by weakening the link between parental income and education investment. Given the unique role of early education and the wide funding differences in this area between the U.S. and some European countries, this paper assesses quantitatively the impact of changes to public early education funding on persistence.

We build a dynamic stochastic life-cycle model where human capital is accumulated throughout early childhood and two periods of later childhood. We calibrate the model to match U.S. earnings and education spending statistics. Under our calibration, early education spending is more productive than, and complementary with, later spending. These two features are consistent with studies of early childhood. We then assign the high level of early education spending seen in Denmark and Norway to our model of the U.S. economy. We show that this level of spending would eliminate only about 8 percent of the persistence gap between the U.S. and these Nordic countries. Our results indicate that early education funding differences play a limited role in explaining the observed persistence gap. Policy makers apparently must look beyond childhood education funding to achieve large changes in intergenerational persistence. Since government spending is most effective in raising education investment for agents at the low end of the income distribution, we consider progressive spending on early childhood. Our results imply a larger change in persistence from progressive spending than from broad-based spending.

Prior work such as that by Restuccia and Urrutia (2004) and Holter (2014), considers the effect of public spending on persistence. The size and direction of the policy effects in our paper confirm the findings from related work. Through our focus on the distinction between early and

\textsuperscript{11}See Corak (2006).
later childhood, we also extend these findings. While we have provided a rich environment for this investigation, further extensions may provide additional insights. One feature missing from our model is parental time investment. We argue that for considering the policies in our paper, this abstraction is warranted. However, with its inclusion, a wider set of policies such as parental leave policy could be analyzed.
References


